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Localization of Cesaro Means of Fourier Series for Functions of Bounded Λ-Variation

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Abstract—We consider classes of periodic functions of bounded Λ-variation, where Λ has a power growth rate. We show that this class contains a continuous function whose Cesaro means of the Fourier series (whose order depends on the growth rate of Λ) have no localization property.

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Introduce the necessary denotations. Put $\mathbb{T} = [-\pi, \pi]$. Let the symbol $C(\alpha)$ stand for various values that depend only on α (possibly different in different cases). For an interval I on the axis the symbol $\Omega(I)$ stands for the set of all finite systems of pairwise disjoint intervals $\{I_n\}$ such that $\overline{I_n} \subset I$, the symbol $\chi_I(t)$ denotes an indicator of I, and $f(I)$ does the increment of the function f on I.

We say that a non-decreasing sequence of positive values $\Lambda = \{\lambda_n\}$ defines a class of functions of bounded Λ-variation (a Waterman class) if $\sum\limits_{n=1}^{\infty} \frac{1}{\lambda_n} = \infty$. In what follows we consider only such sequences Λ. Denote the partial sums $\sum\limits_{k=1}^{N} \frac{1}{\lambda_k}$ by $\Lambda(N)$. Put $H = \{n\}_{n=1}^{\infty}$.

Definition 1. Let Λ be a sequence defining a Waterman class. Then the Λ-variation of a function $f(x)$ over an interval Δ is the value

$$V_\Lambda^x(f;\Delta) = \sup_{\{I_k\}\in\Omega(\Delta)} \sum_k \frac{|f(I_k)|}{\lambda_k}.$$

We call the set of functions, for which this value is finite, the class of bounded Λ-variation on Δ; we denote it by $\Lambda BV(\Delta)$.

Classes ΛBV were defined in the one-dimensional case by D. Waterman in [1], where he has proved the following theorem.

Theorem A. *Let $f \in HBV(\mathbb{T})$ be a 2π-periodic function. Choose an arbitrary point $x \in \mathbb{T}$. Then the Fourier series of the function f converges at x to the value $\frac{1}{2}(f(x+0)+f(x-0))$, and the convergence is uniform on any segment lying inside the interval of continuity of this function. Each class $\Lambda BV(\mathbb{T})$ not embedded into $HBV(\mathbb{T})$ contains a continuous function whose Fourier series diverges at a point.*

Let us recall the definition of Cesàro summation methods (e.g., [3], Chap. 3, § 1). Let a value $\alpha > -1$ be given. Numbers A_n^α are defined by the equality

$$\sum_{n=0}^{\infty} A_n^\alpha x^n = (1-x)^{-\alpha-1}, \text{ i.e., } A_n^\alpha = \frac{(\alpha+1)\dots(\alpha+n)}{n!}.$$

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