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Solvability of Some Classes of Nonlinear Integro-Differential Equations with Noncompact Operator

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Abstract—For a class of nonlinear integrodifferential equations with a noncompact Urysohn-type operator we prove the existence of nonnegative bounded solutions. We study the asymptotic behavior of solutions at infinity. We give some examples that are of practical interest.

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1. INTRODUCTION

In the theory of integral equations, nonlinear problems have a particular place due to their complexity and importance. These problems, besides their particular mathematical interest, represent certain interest in the theory of radiative transport in spectral lines, in the kinetic theory of gases, and in econometrics [1–3].

Nonlinear integral equations of the Urysohn type were mainly studied on finite intervals under the assumption that the Urysohn operator and its linear minorant are completely continuous [4]. Recently one of the authors of this paper considered the Urysohn integral equation without the assumption of the complete continuity of the initial nonlinear operator and proved the existence of a nonnegative bounded solution [5, 6]. In this paper we show that the approach and methods proposed in [5, 6] are applicable to studying some classes of nonlinear integro-differential equations of the Urysohn and Hammerstein types.

Consider the following nonlinear integro-differential equation:

$$\frac{dy(x)}{dx} + qy(x) = \int_0^{+\infty} K(x,t,y(t))dt; \quad x > 0, \tag{1}$$

subject to

$$y(0) = y_0 \geq 0, \tag{2}$$

where $q > 0$ is a positive number, while $K(x,t,\tau)$ is a given nonnegative function defined on the set $R^+ \times R^+ \times R$ that satisfies certain conditions (see Theorems 1–3). In particular, for $K(x,t,y) = K_0(x,t)y$, where K_0 is a nonnegative measurable function satisfying certain conditions, Eq. (1) is studied in papers [7–9].

We conditionally call problem (1)–(2) *homogeneous*, if $y_0 = 0$, and *inhomogeneous*, if $y_0 > 0$.

We seek for a solution to Eq. (1) in the class

$$P = \{f(x) : f(x) \in AC_{\rm loc}[0,+\infty),\ \lim_{x\to+\infty} e^{-\varepsilon x} f(x) = 0\ \forall\varepsilon > 0\} \tag{3}$$

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