Gravitational Interaction and physical spacetime geometry

A. V. Minkevich
Belarussian State University
Warmia and Mazury University in Olsztyn

Abstract

Satisfying the correspondence principle with general relativity theory (GR) in linear approximation with respect to metric and torsion tensors, the gravitation theory in Riemann-Cartan spacetime leads to the change of gravitational interaction in comparison with GR and Newton’s gravitation theory by certain conditions and allows to solve the problem of gravitational singularity in cosmology and also to explain the acceleration of cosmological expansion at present epoch as result without any dark energy.

[1] A.V. Minkevich, Limiting energy density and a regular accelerating Universe in Riemann-Cartan spacetime, *JETP Letters*, 94, No 12, 831- 836 (2011).

[2] A.V. Minkevich, De Sitter spacetime with torsion as physical spacetime in the vacuum and isotropic cosmology, *Mod. Phys. Lett. A* 26, No 4, 259-266 (2011).

[3] А.В. Минкевич, Калибровочный подход в теории тяготения, физическое пространство-время и гравитационное взаимодействие, *Постранство, время и фундаментальные взаимодействия*, вып. 1, 62-72 (2012).

[4] A.V. Minkevich, A.S. Garkun, V.I. Kudin, On some physical aspects of isotropic cosmology in Riemann-Cartan spacetime, *Journal of Cosmology and Astroparticle Physics*, 03, 40 (2013).

[5] A.V. Minkevich, On theory of regular accelerating Universe in Riemann-Cartan spacetime, *Mod. Phys. Lett. A* 28, No 21 (2013) 1350090 (12 p.)

1. Gravitation theory in Riemann-Cartan spacetime (Poincaré gauge theory of gravity – PGTG) as a necessary generalization of metric theory of gravity in the framework of gauge approach by including of the Lorentz group to the gauge group corresponding to gravitational interaction.

Poincaré Gauge Theory of Gravity as natural and necessary generalization of General Relativity Theory

Gauge gravitaional field variables:

h^i_μ – the tetrad, A^{ik}_μ – the Lorentz connection.

Gauge gravitaional field strengths:

$F^{ik}_{\mu\nu}$ – the curvature tensor, $S^i_{\mu\nu}$ – the torsion tensor.

Gravitational Lagrangian:

$$L_g = f_0 F + F^{\alpha\beta\mu\nu}\left(f_1 F_{\alpha\beta\mu\nu} + f_2 F_{\alpha\mu\beta\nu} + f_3 F_{\mu\nu\alpha\beta}\right) + F^{\mu\nu}\left(f_4 F_{\mu\nu} + f_5 F_{\nu\mu}\right) + f_6 F^2$$

$$+ S^{\alpha\mu\nu}\left(a_1 S_{\alpha\mu\nu} + a_2 S_{\nu\mu\alpha}\right) + a_3 S^{\alpha}{}_{\mu\alpha} S_{\beta}{}^{\mu\beta} \quad \left(F_{\mu\nu} = F^{\alpha}{}_{\mu\alpha\nu},\ F = F^{\mu}{}_{\mu}\right),\ f_0 = (16\pi G)^{-1}.$$

Gravitational equations of PGTG

$$\frac{1}{h}\frac{¢\left(h L_g\right)}{¢\ h^i{}_«} = T_i{}^«,\quad \frac{1}{h}\frac{¢\left(h L_g\right)}{¢\ A^{ik}{}_«} = -J_{ik}{}^« \quad \left(h = \det\left(h^i{}_«\right) = \sqrt{-g}\right),$$

where

$$T_i{}^« = -\frac{1}{h}\frac{¢\left(h L_m\right)}{¢\ h^i{}_«}$$ - energy-momentum tensor

$$J_{ik}{}^« = \frac{1}{h}\frac{¢\left(h L_m\right)}{¢\ A^{ik}{}_«}$$ - spin momentum tensor

(L_m – matter Lagrangian)

$$\nabla_\nu U_i{}^{\mu\nu} + 2S^k{}_{i\nu} U_k{}^{\mu\nu} + 2(f_0 + 2f_6\, F)F^\mu{}_i$$
$$+4f_1\, F_{klim} F^{kl\mu m} + 4f_2\, F^{k[m\mu]l} F_{klim}$$
$$+4f_3\, F^{\mu klm} F_{lmik} + 2f_4 (F_{ki} F^{k\mu} + F^\mu{}_{kim} F^{km})$$
$$+2f_5 (F_{ki} F^{\mu k} + F^\mu{}_{kim} F^{mk}) - h_i{}^\mu \mathcal{L}_g = -T_i{}^\mu,$$

$$4\nabla_\nu [(f_0 + 2f_6\, F) h_{[i}{}^\nu h_{k]}{}^\mu + f_1\, F_{ik}{}^{\nu\mu}$$
$$+f_2\, F_{[i}{}^{[\nu}{}_{k]}{}^{\mu]} + f_3\, F^{\nu\mu}{}_{ik} + f_4\, F_{[k}{}^{[\mu} h_{i]}{}^{\nu]} +$$
$$+f_5\, F^{[\mu}{}_{[k} h_{i]}{}^{\nu]}] + U_{[ik]}{}^\mu = J_{[ik]}{}^\mu,$$

$$U_i{}^{\mu\nu} = 2(a_1\, S_i{}^{\mu\nu} - a_2\, S^{[\mu\nu]}{}_i - a_3\, S_\alpha{}^{\alpha[\mu} h_i{}^{\nu]}).$$

2. PGTG based on general expression of gravitational Lagrangian including both a scalar curvature and quadratic in the curvature and torsion invariants and fulfilment of correspondence principle with general relativity theory (GR) in linear approximation in the metric and torsion tensors.

$$G^{(1)}_{\text{«¬}} = \frac{1}{2 f_0} T^{\text{sym}}_{\text{«¬}} + \sim (\text{¦}_{\text{«¬}} \square - \partial_{\text{«}} \partial_{\text{¬}}) T$$

$$g_{\text{«¬}} = \text{¦}_{\text{«¬}} + h_{\text{«¬}}$$

$$2a_1 + a_2 + 3a_3 = 0$$

$$4(f_1 + \frac{1}{2} f_2 + f_3) + f_4 + f_5 = 0$$

$$\sim = \frac{f}{3 f_0^2}, \quad f = f_1 + \frac{1}{2} f_2 + f_3 + f_4 + f_5 + 3 f_6 > 0$$

3. Gravitational interaction at cosmological scale and regular isotropic cosmology of accelerating Universe in Riemann-Cartan spacetime. Gravitational repulsion effect at extreme conditions and vacuum repulsion effect.

Homogeneous isotropic models (HIM) in PGTG

Metrics:

$$g_{\text{«¬}} = \mathrm{diag}\left(1, -\frac{R^2(t)}{1-kr^2}, -R^2(t)r^2, -R^2(t)r^2\sin^2 \text{—}\right)$$

Torsion:

$S^{1}{}_{10} = S^{2}{}_{20} = S^{3}{}_{30} = S_1(t),$

$S_{123} = S_{231} = S_{312} = S_2(t)\ (R^3 r^2/\sqrt{1-kr^2}\ )\ \sin\theta$

Cosmological equations for HIM

$$\frac{k}{R^2} + (H - 2S_1)^2 - S_2^2 = A_2,$$

$$\dot{H} + H^2 - 2HS_1 - 2\dot{S}_1 = A_1,$$

$$a = 2a_1 + a_2 + 3a_3, \qquad b = a_2 - a_1,$$

$$f = f_1 + \frac{f_2}{2} + f_3 + f_4 + f_5 + 3f_6,$$

$$q_1 = f_2 - 2f_3 + f_4 + f_5 + 6f_6, \qquad q_2 = 2f_1 - f_2.$$

In order to exclude higher derivatives of *R* from cosmological equations we have to put a=0; then cosmological equations contain 4 indefinite parameters:

$$\sim \; = \frac{f}{3 f_0^{2}}, \; b, \; \pounds \; = \frac{q_2}{f}, \; \mu \; = \frac{q_1 + q_2 - 2f}{f}$$

$$
\begin{aligned}
\frac{k}{R^2} + (H - 2S_1)^2 = \\
\frac{1}{6f_0Z}\left[\rho + 6\left(f_0Z - b\right)S_2^2 + \frac{\alpha}{4}\left(\rho - 3p - 12bS_2^2\right)^2\right] \\
-\frac{3\alpha\varepsilon f_0}{Z}\left[\left(HS_2 + \dot{S}_2\right)^2 + 4\left(\frac{k}{R^2} - S_2^2\right)S_2^2\right],
\end{aligned}
$$

$$
\begin{aligned}
\dot{H} + H^2 - 2HS_1 - 2\dot{S}_1 = \\
-\frac{1}{12f_0Z}\left[\rho + 3p - \frac{\alpha}{2}\left(\rho - 3p - 12bS_2^2\right)^2\right] \\
-\frac{\alpha\varepsilon}{Z}\left(\rho - 3p - 12bS_2^2\right)S_2^2 \\
+\frac{3\alpha\varepsilon f_0}{Z}\left[\left(HS_2 + \dot{S}_2\right)^2 + 4\left(\frac{k}{R^2} - S_2^2\right)S_2^2\right].
\end{aligned}
$$

Equations for torsion functions

$$S_1 = -\frac{\alpha}{4Z}[\dot{\rho} - 3\dot{p} + 12f_0(3\varepsilon - \omega)HS_2^2$$
$$-12(2b - (\varepsilon - \omega)f_0)S_2\dot{S}_2],$$

$$\varepsilon[\ddot{S}_2 + 3H\dot{S}_2 + \left(3\dot{H} - 4\dot{S}_1 + 12HS_1 - 16S_1^2\right)S_2]$$
$$-\frac{1}{3f_0}[(1 + \frac{1}{2}\omega)(\rho - 3p - 12bS_2^2)$$
$$+\frac{(1 - b/f_0)}{\alpha} - 6f_0\omega A_2]S_2 = 0,$$

$$Z = 1 + \alpha\left(\rho - 3p - 12\left(b + \varepsilon f_0\right)S_2^2\right)$$

HIM at $\varepsilon = 0, \omega \neq 0$

$$\frac{k}{R^2} + (H - 2S_1)^2 =$$

$$\frac{1}{6f_0Z}\left[\rho + 6\left(f_0Z - b\right)S_2^2 + \frac{\alpha}{4}\left(\rho - 3p - 12bS_2^2\right)^2\right]$$

$$\dot{H} + H^2 - 2HS_1 - 2\dot{S}_1 =$$

$$-\frac{1}{12f_0Z}\left[\rho + 3p - \frac{\alpha}{2}\left(\rho - 3p - 12bS_2^2\right)^2\right].$$

Equations for torsion functions

$$S_1 = -\frac{\alpha}{4Z}[\dot{\rho} - 3\dot{p} + 12 f_0 \omega H S_2^2 - 12(2b - \omega f_0) S_2 \dot{S}_2],$$

$$S_2^2 = \frac{\rho - 3p}{12b} + \frac{1 - (b/2f_0)(1 + \sqrt{X})}{12b\alpha(1 - \omega/4)},$$

$$X = 1 + \omega(f_0^2/b^2)[1 - (b/f_0) - 2(1 - \omega/4)\alpha(\rho + 3p)]$$

HIM and limiting energy density

If $0<\omega<4$ ($0<\omega\alpha<<1$), from condition $X\geq0$ follows the restriction on admissible values of energy density:

$$X = 1 + \omega(f_0^2/b^2)[1-(b/f_0) - 2(1-\omega/4)\alpha(\rho+3p)] \geq 0$$

The order of limiting energy density is $(\omega\alpha)^{-1}$

Inflationary cosmological HIM

$$\rho = \frac{1}{2}\dot{\phi}^2 + V + \rho_m \quad (\rho > 0),$$

$$p = \frac{1}{2}\dot{\phi}^2 - V + p_m.$$

$$\dot{\rho}_m + 3H\left(\rho_m + p_m\right) = 0,$$

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\partial V}{\partial \phi}.$$

Dimensionless parameters

$$t \to \tilde{t} = t/\sqrt{6 f_0 \omega\alpha}, \qquad R \to \tilde{R} = R/\sqrt{6 f_0 \omega\alpha},$$

$$\rho \to \tilde{\rho} = \omega\alpha\, \rho, \qquad p \to \tilde{p} = \omega\alpha\, p,$$

$$\phi \to \tilde{\phi} = \phi/\sqrt{6 f_0}, \qquad b \to \tilde{b} = b/f_0,$$

$$H \to \tilde{H} = \tilde{R}'/\tilde{R} = H\sqrt{6 f_0 \omega\alpha}, \quad V \to \tilde{V} = \omega\alpha V,$$

$$S_1 \to \tilde{S}_1 = S_1\sqrt{6 f_0 \omega\alpha}, \qquad S_2 \to \tilde{S}_2 = S_2\sqrt{6 f_0 \omega\alpha},$$

Equations in dimensionalless form

$$\tilde{S}_2^2 = \frac{\tilde{\rho} - 3\tilde{p}}{2\tilde{b}} + \omega \frac{1 - (\tilde{b}/2)(1 + \sqrt{X})}{2\tilde{b}(1 - \omega/4)}; \qquad \tilde{S}_1 = -\frac{3}{4\tilde{b}Z}(\tilde{H}\tilde{D} + \tilde{E}),$$

$$\tilde{D} = \frac{1}{2}\left(3\frac{d\tilde{p}_m}{d\tilde{\rho}_m} - 1\right)(\tilde{\rho}_m + \tilde{p}_m) + \frac{1}{3}(\tilde{\rho}_m - 3\tilde{p}_m) + \frac{2}{3}\tilde{\phi}'^2 + \frac{4}{3}\tilde{V} - \frac{\omega\tilde{b}}{6(1 - \omega/4)}\sqrt{X}$$
$$+ \frac{1 - (\omega/2\tilde{b})}{2\sqrt{X}}\left[\left(3\frac{d\tilde{p}_m}{d\tilde{\rho}_m} + 1\right)(\tilde{\rho}_m + \tilde{p}_m) + 4\tilde{\phi}'^2\right] + \frac{\omega(1 - \tilde{b}/2)}{3(1 - \omega/4)},$$

$$\tilde{E} = \left(1 + \frac{1 - (\omega/2\tilde{b})}{\sqrt{X}}\right)\frac{\partial\tilde{V}}{\partial\tilde{\phi}}\tilde{\phi}',$$

$$\frac{k}{\tilde{R}^2} + \left[\tilde{H}(1 + \frac{3}{2\tilde{b}Z}\tilde{D}) + \frac{3}{2\tilde{b}Z}\tilde{E}\right]^2 = \frac{1}{Z}\Bigg[\tilde{\rho} + (1/2)(\frac{Z}{\tilde{b}} - 1)$$

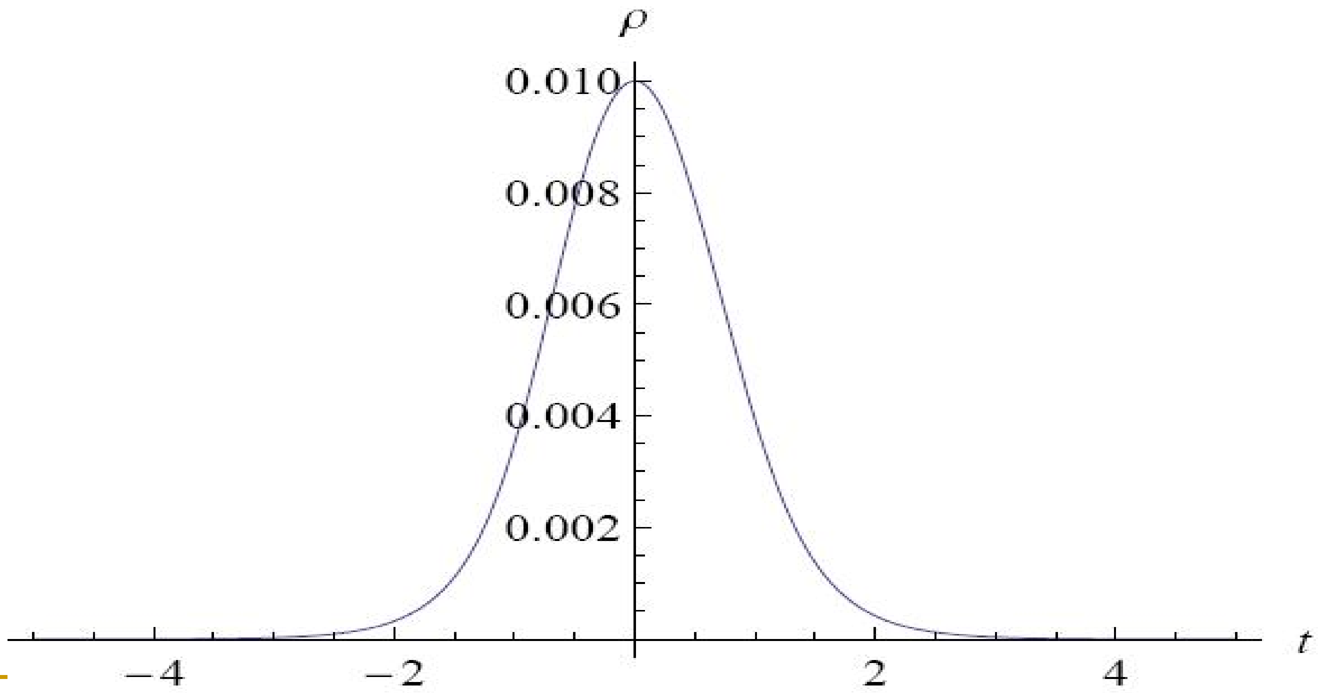
$$\left[\tilde{\rho} - 3\tilde{p} + \omega\frac{1 - (\tilde{b}/2)(1 + \sqrt{X})}{1 - \omega/4}\right] + \omega\frac{[1 - (\tilde{b}/2)(1 + \sqrt{X})]^2}{4(1 - \omega/4)^2}\Bigg],$$

$$(\tilde{H}' + \tilde{H}^2)\left(1 + \frac{3}{2\tilde{b}Z}\tilde{D}\right) + \frac{3}{2\tilde{b}Z}\left[\tilde{H}(\tilde{D}' - \frac{Z'}{Z}\tilde{D} + \tilde{E}) + E' - \frac{Z'}{Z}\tilde{E}\right]$$

$$= -\frac{1}{2Z}\left[\tilde{\rho} + 3\tilde{p} - \omega\frac{[1 - (\tilde{b}/2)(1 + \sqrt{X})]^2}{2(1 - \omega/4)^2}\right].$$

$$p_m = \frac{1}{3}\rho_m \qquad V = m^2\phi^2/2$$

$$b = 0.999,\ \omega = 10^{-8},\ m = 0.1,\ H = 0,\ \phi_0 = 30,\ \rho_0 = 0.01,\ \phi_0' = -2.15$$



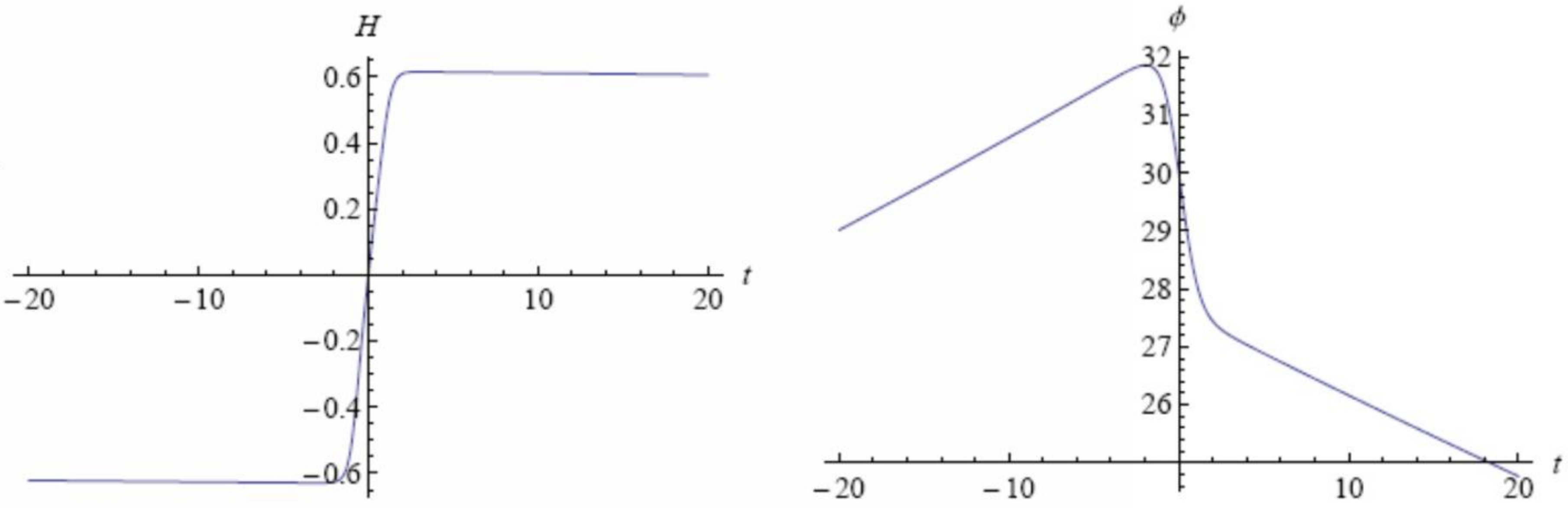


Figure 1. Transition stage.

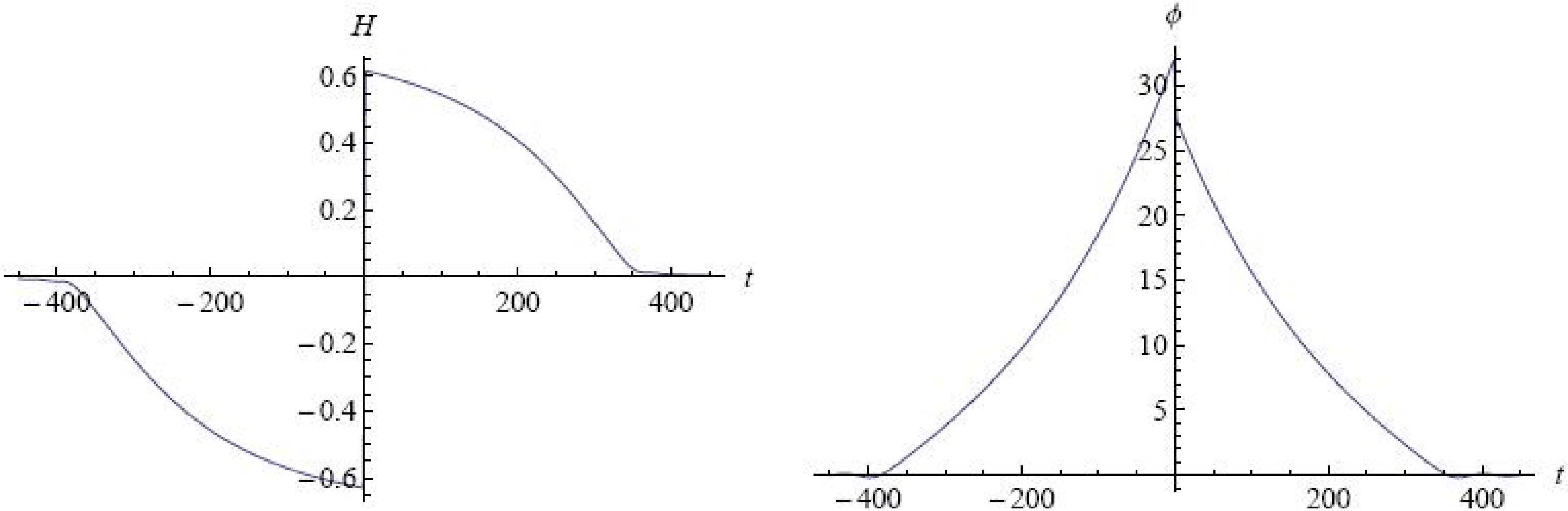


Figure 2. Inflationary stage.

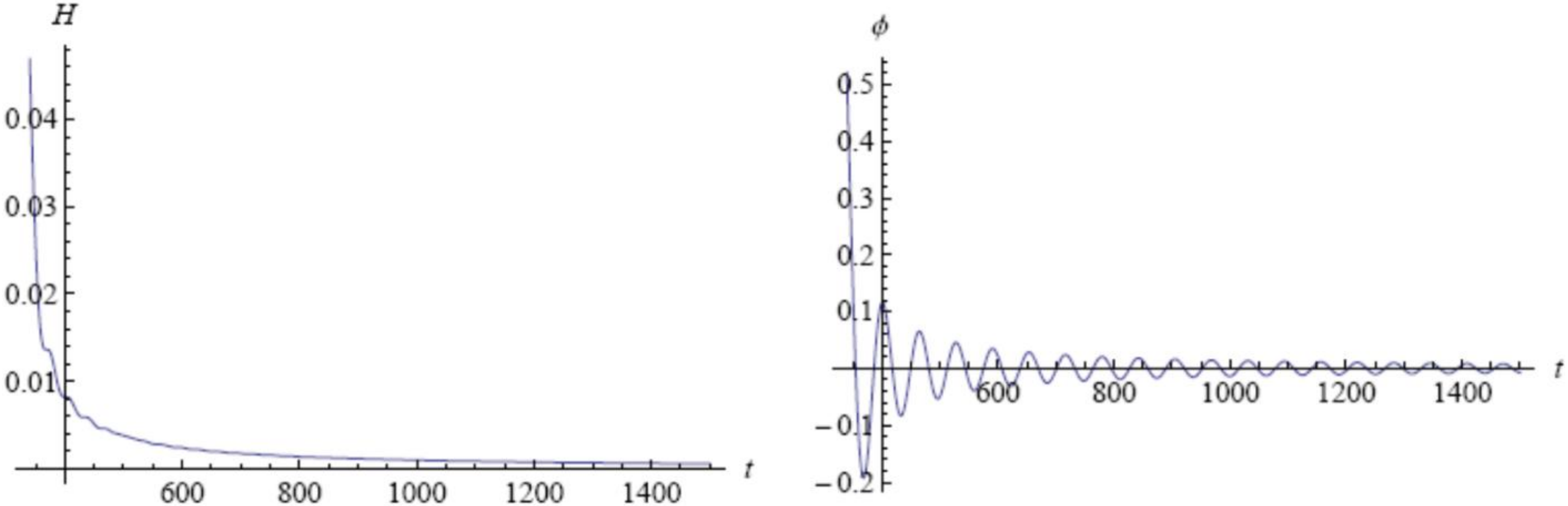


Figure 3. Postinflationary stage.

Cosmological equations with two torsion functions at asymptotics

If $|\omega|<<1$ or $\omega\sim1$ and $0<1-(b/f_0)<<1$ we obtain at asymptotics:

$$S_2^2 = \frac{\rho - 3p}{12b} + \frac{1 - b/f_0}{12\alpha b}$$

$$\frac{k}{R^2} + H^2 = \frac{1}{6f_0}\left[\rho(f_0/b) + \frac{1}{4}\alpha^{-1}(1 - b/f_0)^2(f_0/b)\right]$$

$$\dot{H} + H^2 = -\frac{1}{12f_0}\left[(\rho + 3p)(f_0/b) - \frac{1}{2}\alpha^{-1}(1 - b/f_0)^2(f_0/b)\right]$$

Comparison with Friedmann cosmological equations of GR

$$\frac{k}{R^2} + H^2 = \frac{1}{6f_0}\rho_{tot},$$

$$\dot{H} + H^2 = -\frac{1}{12f_0}(\rho_{tot} + 3p_{tot}),$$

$$\circ_{tot} = \circ_B + \circ_{DM} + \circ_{DE}, \qquad p_{tot} = p_B + p_{DM} + p_{DE}$$

$$p_B = p_{DM} = 0, \qquad p_{DE} = -\circ_{DE}.$$

Cosmological equations of PGTG at asymptotics coincide with Friedmann cosmological equations if $b = f_0\left(\circ_0/(\circ_0 + \circ_{DM\,0}\right), \quad \sim = \circ_{DE\,0}^{(-1)}\left(1 - b/f_0\right)^2\left(f_0/b\right)$

Vacuum spacetime in PGTG

$$H^2\left[1+\frac{2(2f-q_1+2q_2)}{(f_0+a/8)Z}S_2^2\right]^2 - S_2^2 = A_2$$

$$4q_2S_1(3H-4S_1) - \frac{q_1+q_2}{3}F + 2(q_1+q_2-2f)A_2 - (f_0-b) = 0$$

$$F = \frac{6}{f_0+a/8}\left[-(b+a/8)S_2^2 + \frac{a}{4}H^2\right]$$

$$S_1 = -\frac{2f-q_1+2q_2}{(f_0+a/8)Z}HS_2^2\,,$$

$$A_2 = \frac{1}{6(f_0+a/8)Z}\left[-6(b+a/8)S_2^2 + \frac{f}{3}F^2 + \frac{3a}{4}H^2 - 6q_2(H^2-4S_2^2)S_2^2\right]$$

4. Regular massive objects in galaxies as alternative to singular black holes of GR.
5. Gravitational interaction at astrophysical scale and dark matter problem of GR.
6. Some unsolved problems of PGTG.

THANK YOU!