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ITERATION METHODS OF SOLVING VARIATIONAL INEQUALITIES OF THE SECOND KIND WITH INVERSE STRONGLY MONOTONE OPERATORS

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Introduction

In this article we suggest iteration methods of solving variational inequalities of the second kind with inverse strongly monotone operators and convex non-differentiable functionals. Inequalities of that sort arise, for example, in the description of processes of stationary filtration, problems of determination of the equilibrium of soft shells. As a preliminary, we consider the problems on search of saddle points of the Lagrange function and the modified Lagrange function. We establish a connection between the solutions of these problems and the initial problem. This makes it possible to apply for the construction of an iteration method of solving the considered variational inequality the algorithm of finding a saddle point of the modified Lagrange function. We investigate the convergence of the suggested method. In the capacity of an example, we consider stationary problems of filtration of an incompressible liquid, which obeys a discontinuous law of filtration with a limit gradient.

1. Statement of problem

Let V and H be Hilbert spaces with the scalar products $(\cdot,\cdot)_V$ and $(\cdot,\cdot)_H$, respectively; $F(\eta) = \Psi(\eta) + G(\Lambda\eta)$, $\Psi : V \to R^1$ be a Gateaux-differentiable functional and its Gateaux derivative $A = \Psi' : V \to V$ be a coercive and inverse strongly monotone operator, i. e., (see [1])

$$(Au - A\eta,\, u - \eta)_V \geq \sigma \|Au - A\eta\|_V^2, \quad \sigma > 0 \ \ \forall u, \eta \in V; \tag{1}$$

$G : H \to R^1$ be a proper, convex, continuous functional; $\Lambda : V \to H$ be a linear continuous operator possessing the bounded inverse such that $(\Lambda u, \Lambda\eta)_H = (u, \eta)_V \ \forall u, \eta \in V$.

We consider the problem

$$F(u) = \inf_{\eta \in V} F(\eta). \tag{2}$$

This problem is equivalent (see, e. g., [2]) to the following variational inequality of the second kind

$$(Au,\, \eta - u)_V + G(\Lambda\eta) - G(\Lambda u) \geq 0 \ \ \forall \eta \in V \tag{3}$$

and has at least one solution. Variational inequalities of that sort arise, in particular, in the description of processes of stationary filtration, problems of determination of equilibrium state of soft shells (see [3], [4]).

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