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On Continuation of a Topology from a System of Generators of a Group to a Topology on the Group

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Abstract—In this paper we establish conditions under which a topology is continued from a subset of a group to a topology on the group coordinated with the group operation.

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The problem of topological embedding of a semigroup with a topology to a group with a topology as an open subsemigroup is equivalent to the problem of continuation of a topology with a subsemigroup to a topology on a group under the assumption that the topology on the semigroup is induced by the topology on the group. Under certain assumptions about the consistency of the topology on a group with the group operation, the problem becomes nontrivial.

In [1] F. Cristoph establishes the necessary and sufficient condition for the topological embedding of a topological semigroup (algebraically embedded into a group) in a topological group. However, the verification of the mentioned condition is rather difficult.

Let semitopological semigroups with cancellations be commutative [2] or reversing [3]. If a topology on these semigroups is locally compact and all translations are open, then there is a possibility of their topological embedding in locally compact groups.

In [4] a similar result is obtained for arbitrary semitopological semigroups with open translations. Let us note the paper [5], where one formulates the necessary and sufficient conditions for the embedding of a semitopological semigroup into a semitopological group as an open subsemigroup (theorem 2.1). However, the proof of this result in [5] is incomplete because the inductive assumption on the length of the product of elements of $S \cap S^{-1}$, where S is a subsemigroup of a group G and S generates the group G, is unproved.

In this paper we generalize results of [4] for the case of a generating subset of a group with a topology defined on this subset.

Recall some definitions used in this paper.

A group with a topology is called *a topological group* if the group operation is continuous over the set of all arguments and the inversion operation is continuous.

Let X be a semigroup and $a \in X$. Then mappings $\rho_a(x) = xa$ and $\lambda_a(x) = ax$ are called, respectively, the *right inner translation* and the *left* one of the semigroup X on the element a.

A semigroup (group) with a topology is called a *semitopological semigroup* (*group*), if each left translation and each right one are continuous mappings.

A set X with an associative n-ary operation ( ) is called an n-semigroup and denoted as $(X, (\ ))$.

In the proof of the following theorem we use the well-known **Ellis theorem**: Let X be a group endowed with a locally compact topology with continuous right and left inner translations. Then X is a topological group [6].

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