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MULTIPOINT INVARIANTS OF TRANSFORMATION GROUPS AND THREE-WEB DEFINED BY THEM

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Introduction. The three-webs determined by foliations of different dimensions were first considered in [1]. The structure theory of such webs was developed in [2]. In [3] there are considered some special classes of webs defined by relations on the tensors involved in the structure equations of webs. In [4]–[6], we extended the basic concepts of classical theory to the webs $W(p,q,p+q-1)$, i. e., introduced an analog of coordinate loop of a web, and generalized the Reidemeister configuration. The class of webs for which the generalized Reidemeister configurations are enclosed, is a geometrical equivalent of so-called binary physical structures [7]. The webs WR are characterized by a special relation on parameters of leaves of the third foliation involved in an arbitrary generalized configuration R. In [4] this relation is called a core since it generalizes the corresponding notion of the theory of classical Bol webs introduced in [8]. (In the theory of physical structures the core is called the phenomenologically invariant form of physical law.) In papers devoted to the theory of physical structures (see [9], and also the reference list in [10]), the core is found only for some physical structures corresponding to three-webs whose third foliation consists of hypersubmanifolds. At the same time, in these papers the geometrical sense of core remains unclear. In [6] it was shown that the core of a web defined by a transformation group is an equality between two invariants of group. In the present paper we find cores of webs defined by affine and projective groups.

1. Let us find point invariants of the group of affine transformations of plane. It is impossible to associate an affine invariant with three points of plane which do not lay on a straight line. Let us fix four points on plane y_1, y_2, y_3, and y_4. Let y_5 be the intersection point for diagonals y_1y_3 and y_2y_4 of the quadrangle $y_1y_2y_3y_4$. Let us denote by S_{123} the area of the triangle $y_1y_2y_3$, by S_{124} the area of triangle $y_1y_2y_4$, etc. The ratio of S_{125} and S_{123} is an affine invariant, since this ratio is equal to the ratio of the triangle bases:

$$\frac{S_{125}}{S_{123}}=\frac{y_1y_5}{y_1y_3}.$$

In the same way we find another three affine invariants:

$$\frac{S_{125}}{S_{124}},\quad \frac{S_{345}}{S_{234}},\quad \frac{S_{345}}{S_{134}}.$$

By eliminating S_{125} and S_{345} from these expressions, we get the following invariants:

$$\frac{S_{123}}{S_{124}},\quad \frac{S_{134}}{S_{234}}.$$

Thus, the affine group of plane has two four-point invariants.