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On the Non-Existence of Periodic Orbits for a Class of Two-Dimensional Kolmogorov Systems

R. Boukoucha[1**]

[1] *University of Bejaia, 06000 Bejaia, Algeria*

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Abstract—In this paper we introduce an explicit expression of first integral, then we prove the non-existence of periodic orbits, then consequently the non-existence of limit cycles of two-dimensional Kolmogorov system, where $R(x,y)$, $S(x,y)$, $P(x,y)$, $Q(x,y)$, $M(x,y)$, $N(x,y)$ are homogeneous polynomials of degrees m, a, n, n, b, b, respectively. We introduce concrete example exhibiting the applicability of our result.

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INTRODUCTION

The autonomous differential system on the plane given by

$$\begin{cases} x' = \frac{dx}{dt} = xF(x,y), \\ y' = \frac{dy}{dt} = yG(x,y), \end{cases} \tag{1}$$

known as Kolmogorov system, the derivatives are performed with respect to the time variable, and F, G are two functions in the variables x and y, is frequently used to model the iteration of two species occupying the same ecological niche [1, 2, 3]. There are many natural phenomena which can be modeled by the Kolmogorov systems such as mathematical ecology and population dynamics [4–7] chemical reactions, plasma physics [8], hydrodynamics [9], economics, etc. In the classical Lotka–Volterra–Gause model, F and G are linear and it is well-known that there are no limit cycles. Of course, there can be only one critical point in the interior of the realistic quadrant ($x > 0$, $y > 0$) in this case, but this can be a center; however, there are no isolated periodic solutions. We recall that in the phase plane, a limit cycle of system (1) is an isolated periodic orbit in the set of all periodic orbits of system (1). In the qualitative theory of planar dynamical systems [10–15], one of the most important topics is related to the second part of the unsolved Hilbert 16th problem. There is a huge literature about limit cycles, most of them deal essentially with their detection, their number and their stability and rare are papers concerned by giving them explicitly [16–18].

System (1) is integrable on an open set Ω of $\mathbb{R}^2$ if there exists a non-constant C^1 function $H : \Omega \to \mathbb{R}$, called a first integral of the system on Ω, which is constant on the trajectories of the system (1) contained in Ω, i.e., if

$$\frac{dH(x,y)}{dt} = \frac{\partial H(x,y)}{\partial x} xF(x,y) + \frac{\partial H(x,y)}{\partial y} yG(x,y) \equiv 0 \text{ in the points of } \Omega.$$

Moreover, $H = h$ is the general solution to this equation, where h is an arbitrary constant. It is well-known that for differential systems defined on the plane $\mathbb{R}^2$ the existence of a first integral determines their phase portrait [19].

*The text was submitted by the author in English

**E-mail: arachid_boukecha@yahoo.fr.