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ON THE STABILITY OF THE SOLUTION OF THE PROBLEM ON THE BEST APPROXIMATION OF A CONVEX COMPACT BY A BALL

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1. Introduction

Let D be a convex compact from a finite-dimensional real space $\mathbb{R}^p$, $\|x\|$ the Euclidean norm of $x \in \mathbb{R}^p$, $h(A,B)$ the Hausdorff distance between sets A and B from $\mathbb{R}^p$, $B(x,r) = \{y \in \mathbb{R}^p : \|x-y\| \le r\}$.

The problem of the best approximation of a convex compact D by a Euclidean ball in the Hausdorff metric can be written in the form

$$h(D, B(x,r)) \to \min_{x\in\mathbb{R}^p,\ r>0}. \tag{1.1}$$

This problem was posed and studied in [1]. As it turned out in particular, problem (1.1) is directly related to another problem concerning the estimate of a compact D. Namely, it was shown that the ball of the best approximation for D is unique, and its center x_0 is contained in D and is the unique solution of the problem

$$\Phi(x) \equiv R(x) - \rho(x) \to \min_{x\in D}, \tag{1.2}$$

where $R(x) = \max\limits_{y\in D} \|x-y\|$, $\rho(x) = \min\limits_{y\in\Omega} \|x-y\|$, $\Omega = \overline{\mathbb{R}^p \setminus D}$. In addition, the radius of the desired ball is $r_0 = (R(x_0) + \rho(x_0))/2$ and

$$h_0(D) \equiv \min_{x\in\mathbb{R}^p,\ r\ge 0} h(D, B(x,r)) = (R(x_0) - \rho(x_0))/2. \tag{1.3}$$

From the geometrical point of view, (1.2) is the problem of constructing a spherical layer of the least width containing the boundary of a convex compact D. As was mentioned in [1], problem (1.2) and similar problems were considered in [2]–[10]. In [11], [12], the authors considered a generalization of problem (1.1) to the case when the Euclidean norm is replaced by an arbitrary norm and the approximation is performed by a ball of this norm in the generated Hausdorff metric.

In this paper, we study the stability of the solution of problem (1.1) with respect to the error of definition of the approximated compact. The aim of the paper is to obtain an estimate of the stability of the center of the ball of the best approximation. Let D_ε be a convex compact such that

$$h(D, D_\varepsilon) \le \varepsilon, \tag{1.4}$$

where $\varepsilon > 0$. As was shown in [1], the function $h_0(\cdot) : Kv(\mathbb{R}^p) \to \mathbb{R}^1_+$ defined by (1.3) on the metric space $Kv(\mathbb{R}^p)$ of all convex compacts from $\mathbb{R}^p$ with Hausdorff metric is Lipschitzian:

$$|h_0(D_1) - h_0(D_2)| \le h(D_1, D_2) \ \ \forall D_1, D_2 \in Kv(\mathbb{R}^p).$$

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