ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2011, Vol. 55, No. 11, pp. 66–70. © Allerton Press, Inc., 2011.
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Regularization of Linear Difference Equations with Analytic Coefficients and Their Applications

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Received March 1, 2011

Abstract—We propose a regularization method for four-element linear difference equations with analytic coefficients. We study these equations in the class of functions which are holomorphic in the complex plane with a cruciform cut and vanish at infinity. We give several examples illustrating the dependence of the solvability properties of equations on the choice of periodic coefficients. We describe various applications.

DOI: 10.3103/S1066369X11110090

Keywords and phrases: *difference equations, regularization method, Carleman problem, moments of entire functions.*

1. Let D be a plane with a cut along the contour $\Gamma[-1,1]\bigcup[-i,i]$. Introduce functions $\sigma_1(z)=z+1+i$, $\sigma_2(z)=z-1+i$, $\sigma_3(z)=z+1-i$, and $\sigma_4(z)=z-1-i$, and the square D_1 with vertices $\sigma_i(0)$, $j=\overline{1,4}$. Consider the linear difference equation (l. d. e.)

$$(Vf)(z)\equiv\sum_{j=1}^{4}G_j(z)f[\sigma_j(z)]=g(z),\quad z\in D_1, \tag{1}$$

under the following assumptions.

1) We seek for a solution $f(z)$ in the class B of functions which are holomorphic in D and satisfy the equality $f(\infty)=0$. Its boundary values $f^{\pm}(t)$ satisfy the Hölder condition everywhere on Γ excluding points 0, ±1, and $\pm i$, where logarithmic singularities are admitted.

2) Coefficients $G_j(z)$, $j=\overline{1,4}$, are holomorphic in the closure $\overline{D_1}$, and $G_j(t)\neq 0$ for $t\in\partial D_1$. In addition, for $t\in\Gamma$ we have $G_1(t-i)G_4(t+i)=G_2(t-i)G_3(t+i)$ if $\operatorname{Im} t=0$, and $G_1(t-1)G_4(t+1)=G_2(t+1)G_3(t-1)$ if $\operatorname{Re} t=0$.

3) The right-hand side $g(z)$ is holomorphic in the square D_1, and $g^+(t)\in H_\nu(\partial D_1)$.

One can easily verify that standard techniques for solving l. d. e. cannot help in this situation even in the simplest case when all coefficients are constant. Really, the set $\mathbb{C}\setminus\bigcup_{j=1}^{4}\sigma_j^{-1}(\Gamma)$ splits into two connected components; one of them (the square D_1) contains the point $z=0$, while another one contains the point at infinity (see, for instance, [1] for the detailed information on this issue). In other words, correlation (1) is fulfilled on the component D_1, but in general it is not valid on another connected component containing the infinity point. This fact does not depend on whether a domain of holomorphy of functions $G_j(z)$ and $g(z)$ "is larger " than the set $\overline{D_1}$.

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