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Solvability of Geometrically Nonlinear Boundary-Value Problems for the Timoshenko-Type Anisotropic Shells with Rigidly Clamped Edges

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Abstract—In the nonlinear theory of shells all known existence theorems are based on the Kirchhoff—Love model. We prove a new existence theorem using the displacement model proposed by S. P. Timoshenko.

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1. Introduction. The problem. The generalized problem solution. All known by now existence theorems in the nonlinear shell theory were proved within the Kirchhoff—Love model ([1—4] and references therein). But the solvability of nonlinear problems connected with more general models (that are not based on the Kirchhoff—Love hypotheses) still was not established. Academician I. I. Vorovich has referred these issues to unsolved problems in the mathematical theory of shells ([1], P. 349). In this paper we study the solvability of nonlinear boundary-value problems for shallow anisotropic inhomogeneous shells with rigidly clamped edges within the shear model proposed by S. P. Timoshenko. The results obtained in this paper develop those of [5], where one proves the solvability of boundary-value problems under more rigid restrictions imposed on physical-geometric characteristics of a shell and on the forces applied to it.

Consider the following system of equations:

$$\begin{gathered}(DT^{i\lambda})_{\alpha^\lambda} + DG^i_{\lambda\mu}T^{\lambda\mu} + R^i = 0, \quad i = 1,2,\\ (DT^{\lambda\mu}w_{3\alpha^\lambda})_{\alpha^\mu} + (DT^{\lambda 3})_{\alpha^\lambda} + DB_{\lambda\mu}T^{\lambda\mu} + R^3 = 0,\\ (DM^{i\lambda})_{\alpha^\lambda} - DT^{i3} + DG^i_{\lambda\mu}M^{\lambda\mu} + N^i = 0, \quad i = 1,2.\end{gathered} \tag{1}$$

It describes the equilibrium state of elastic shallow anisotropic inhomogeneous shells of the Timoshenko type, where forces T^{ij} and moments M^{ij} are given by formulas

$$T^{ij} = D_0^{ijkn}\gamma^0_{kn} + D_1^{ijkn}\gamma^1_{kn}, \quad M^{ij} = D_1^{ijkn}\gamma^0_{kn} + D_2^{ijkn}\gamma^1_{kn}, \quad i,j = \overline{1,3}; \tag{2}$$

the symbol $(DT^{i\lambda})_{\alpha^\lambda}$ means the differentiation in the variable α^λ; here the sum over repeating Latin indices is taken from 1 to 3, the sum over Greek indices is taken from 1 to 2; R^j ($j = \overline{1,3}$) and N^i ($i = 1,2$) are applied forces; γ^0_{kn} and γ^1_{kn} are components of deformations ([6], pp. 168—170, 269):

$$\begin{gathered}\gamma^0_{jj} = w_{j\alpha^j} - G^\lambda_{jj}w_\lambda - B_{jj}w_3 + w^2_{3\alpha^j}/2, \quad \gamma^0_{12} = w_{1\alpha^2} + w_{2\alpha^1} - 2G^\lambda_{12}w_\lambda - 2B_{12}w_3 + w_{3\alpha^1}w_{3\alpha^2},\\ \gamma^1_{jj} = \nu_{j\alpha^j} - G^\lambda_{jj}\nu_\lambda, \quad \gamma^1_{12} = \nu_{1\alpha^2} + \nu_{2\alpha^1} - 2G^\lambda_{12}\nu_\lambda, \quad \gamma^0_{j3} = w_{3\alpha^j} + \nu_j, \gamma^0_{33} = \gamma^1_{i3} \equiv 0,\\ j = 1,2, \quad i = \overline{1,3},\end{gathered} \tag{3}$$

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