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On One Polynomial Method of Solving Integral Equations of the Third Kind

S. A. Solov'eva[1*]

[1] *Naberezhnye Chelny Institute of Kazan (Volga Region) Federal University, pr. Mira 68/19, Naberezhnye Chelny, 423810 Russia*

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Abstract—We propose and substantiate a special variant of the subdomain-collocation method for the approximate solving integral equations of the third kind in the general case of zeros of the coefficient in the space of distributions.

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We consider a linear integral equations of the third kind (ETK)

$$(Ax)(t) \equiv (Ux)(t) + (Kx)(t) = y(t), \tag{1}$$

where

$$(Ux)(t) \equiv u(t)x(t),\;\; u(t) \equiv t^{p_1}(1-t)^{p_2}\prod_{j=1}^{q}(t-t_j)^{m_j},\;\; (Kx)(t) \equiv \int_0^1 K(t,s)x(s)ds,\;\; t \in I \equiv [0,1],$$

$p_1 \geq 0,\; p_2 \geq 0,\; t_j \in (0,1),\; m_j\; (j = \overline{1,q})$ are integer nonnegative numbers; K and y are known continuous functions, with certain "smoothness" properties of pointwise character, and x is the desired function. The equations of the form (1) arise in many problems of theories of neutron transport, elasticity, particle scattering (e.g., [1] and bibliography therein; [2]). The natural classes of solutions of ETKs are, as a rule, special spaces of distributions. It is very seldom that we can obtain the exact solution to equations of the form (1), so the problem of development of approximate methods of their solving with theoretical substantiation is important and actual.

A number of results in this area was obtained in [3–9]. In [3], the author constructs a complete theory of solvability of the considered equations, as well as suggests and theoretically substantiates the methods of their approximate solving in the space of type D of distributions, based on Dirac delta function, and in partial cases of zeros of the coefficient $u(t)$ in the space of type V, constructed by meas of the functional of a "finite part of Hadamard integral". In [4–9] the authors develop and substantiate a number of methods of exact and approximate solving the equations of the form (1) in the space of type V in the most general case of zeros of the coefficient $u(t)$.

In this paper, based on the ideas and the arguments of the works [3–9], we construct and substantiate in the sense of [10] (Chap. 1) a special variant of subdomain-collocation method of solving the equations of the form (1) in the space of distributions of type V in the general case of zeros of the coefficient $u(t)$. The proposed method well allows the structural properties of the source data, namely, as their order of smoothness increases, the rate of convergence of approximate solutions to the exact solution increases, as well.

*E-mail: solovjeva_sa@mail.ru.