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# INTERPOLATION BY EXTREMAL POINTS OF THE CHEBYSHEV POLYNOMIALS AND APPLICATIONS

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## Introduction

Various mathematical subjects and applications widely use the interpolation of continuous functions at extremal points

$$x_k = x_{k,n} = \cos\frac{k\pi}{n}, \quad k = \overline{0,n}, \quad n \in \mathbb{N}, \tag{0.2}$$

of the Chebyshev polynomials of the I kind

$$T_n(x) = \cos n \arccos x, \quad -1 \leqslant x \leqslant 1, \quad n+1 \in \mathbb{N}. \tag{0.3}$$

Interpolating polynomials have the form (see, e.g., [1], [2]) [1]

$$\mathcal{L}_n(f;x) = \frac{1}{n}\sum_{k=0}^{n}{}'' (-1)^{k+1} f(x_k)\frac{(1-x^2)U_{n-1}(x)}{x-x_k} \equiv \sum_{k=0}^{n} f(x_k) l_k(x), \quad -1 \leqslant x \leqslant 1, \quad n \in \mathbb{N}, \tag{0.4}$$

where $f(x)$ is the given continuous function, and $U_{n-1}(x)$ are the Chebyshev polynomials of the II kind

$$U_{n-1}(x) = \frac{\sin n \arccos x}{\sqrt{1-x^2}}, \quad n \in \mathbb{N}. \tag{0.5}$$

In this paper, we try to study systematically the approximative properties of the interpolation process (0.1)–(0.4) and its applications to the solution of frequently used in practice integral and differential equations

$$\varphi(x) + \int_{-1}^{1} \frac{h(x,t)\varphi(t)dt}{\sqrt{1-t^2}} = f(x), \quad -1 \leqslant x \leqslant 1, \tag{0.6}$$

$$\frac{1}{\pi}\int_{-1}^{1} \frac{\ln|x-t|\varphi(t)dt}{\sqrt{1-t^2}} + \frac{1}{\pi}\int_{-1}^{1} \frac{h(x,t)\varphi(t)}{\sqrt{1-t^2}}dt = f(x), \quad -1 \leqslant x \leqslant 1, \tag{0.7}$$

$$\varphi^{(m)}(x) + \sum_{k=1}^{m} a_k(x)\varphi^{(m-k)}(x) = f(x), \quad -1 \leqslant x \leqslant 1, \quad m \in \mathbb{N}, \tag{0.8}$$

[1] The double stroke at the summation symbol means that the terms with $k = 0$ and $k = n$ should be divided by 2.