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Dirichlet Problem for Lavrent'ev–Bitsadze Equation With Loaded Summands

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Abstract—We study the first boundary-value problem for loaded equation of elliptic-hyperbolic type in rectangular domain. We establish a criterion of uniqueness. A solution to the problem is constructed in the form of the sum of a series. In substantiation of existence of a solution to a problem small denominators appear. We obtain the estimates about a separation from zero of denominators with the corresponding asymptotics. They allow to prove existence of a solution in a class of regular solutions.

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INTRODUCTION

Important problems of Mathematical Physics and Biology, in particular, problems of long-term forecasting and regulation of soil water, problems of heat and mass transfer with the finite rate, motion of a little compressible fluid surrounded by a porous medium, optimal control of agro-ecosystems yield boundary-value problems for loaded partial differential equations [1–4].

Let us consider a loaded solution of the mixed type

$$Lu = \begin{cases} u_{xx} + u_{yy} - b^2 u(x,y) + C_1(y)u(x,0) = 0, & y > 0; \\ u_{xx} - u_{yy} - b^2 u(x,y) + C_2(y)u(x,0) = 0, & y < 0, \end{cases} \tag{1}$$

in a rectangular domain $D = \{(x,y) : 0 < x < l,\ -\alpha < y < \beta\}$, α, β, l, $b \geq 0$ are given positive real numbers, $C_1(y)$ and $C_2(y)$ are given continuous functions.

The Dirichlet problem. Find in the domain D a function $u(x,y)$, which satisfies the following conditions:

$$u(x,\, y) \in C^1(\overline{D}) \cap C^2(D_+ \cup D_-); \tag{2}$$

$$Lu(x,y) \equiv 0, \quad (x,y) \in D_+ \cup D_-; \tag{3}$$

$$u(0,y) = u(l,y) = 0, \quad -\alpha \leq y \leq \beta; \tag{4}$$

$$u(x,\beta) = \varphi(x), \quad u(x,-\alpha) = \psi(x), \quad 0 \leq x \leq l, \tag{5}$$

where $\varphi(x)$, $\psi(x)$ are given sufficiently smooth functions, $\varphi(0) = \varphi(l) = \psi(0) = \psi(l)$, $D_+ = D \cap \{y > 0\}$, $D_- = D \cap \{y < 0\}$.

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