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Functional Differential Inequalities and Estimation of the Cauchy Function of an Equation with Aftereffect

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Abstract—We consider scalar functional differential inequalities that are used to estimate solutions to differential equations with deviating argument. A theorem on positiveness of the Cauchy function of a differential equation with aftereffect is derived from a theorem on a functional differential inequality with nonlinear monotone operator, which is a direct generalization of the simplest classical theorem on a differential inequality. The suggested proofs rely on local properties of continuous functions only.

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Dedicated to Professor N. V. Azbelev
on the occasion of his 90th birthday

Among instruments of the qualitative theory of differential equations, the methods of estimate of solutions based on theorems about differential inequalities are particularly important [1]. These theorems and methods are inherited by the modern theory of functional differential equations ([2], Chap. 10; [3]). The main goal of the present paper is to substantiate methods of estimate for solutions to differential equations with aftereffect, possibly combining the simplicity of substantiation with the elementarity of used means. We prove the theorem about the scalar functional differential inequality resolved with respect to the derivative, from which we deduce the main assertions about inequalities used in investigations of stability and non-oscillation of solutions to scalar functional differential equations. As a corollary of the obtained theorem we consider conditions of positivity of the Cauchy function for the differential equation with aftereffect, which is defined by an isotonic operator.

In the proposed proofs we use local properties of continuous functions. The operator language is used for the generality of reasoning and the brevity of writing, if it keeps the clarity of results. This approach principally differs from the usually applied one, because "excluding the classical Chaplygin theorem about the differential inequality for the equation $\dot{x} = f(t, x)$, assertions about inequalities are proven by one or another reduction of the considered equation to the equation $x = Ax$ with operator A defined on a partially ordered set and possessing the isotonic property: From $x_1 \geq x_2$ it follows that $Ax_1 \geq Ax_2$" ([2], P. 211).

1. THEOREMS ABOUT LINEAR FUNCTIONAL DIFFERENTIAL INEQUALITY

Traditionally studied linear scalar differential equations with aftereffect, which are defined on a real semiaxis and resolved with respect to derivative, can be written in the general form ([2], Chap. 5; [4], Chap. I)

$$\dot{x}(t) + \int_a^t x(s)\, d_s r(t, s) = f(t), \quad t \geq a. \tag{1}$$

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