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# EMBEDDED HYPERSURFACES IN THE SPACES WITH FUNDAMENTAL GROUPS $G_2^n$ AND $COG_2$

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The octave algebra $Ca$ (Cayley algebra) can be represented as the direct sum: $Ca = R\cdot 1 \oplus V$, where $V$ is the orthogonal complement to the unit. $V$ is a seven-dimensional vector space invariant with respect to the actions of the Lie groups $G_2$ and $COG_2$. In $V$ there exists a basis $\{\mathbf{e}_1, \dots, \mathbf{e}_7\}$ with the multiplication table of the classical Cayley algebra [1]:

$$\begin{array}{llll}
[\mathbf{e}_1,\mathbf{e}_2]=\mathbf{e}_3, & [\mathbf{e}_1,\mathbf{e}_3]=-\mathbf{e}_2, & [\mathbf{e}_1,\mathbf{e}_4]=\mathbf{e}_5, & [\mathbf{e}_1,\mathbf{e}_5]=-\mathbf{e}_4,\\
[\mathbf{e}_1,\mathbf{e}_6]=-\mathbf{e}_7, & [\mathbf{e}_1,\mathbf{e}_7]=\mathbf{e}_6, & [\mathbf{e}_2,\mathbf{e}_3]=\mathbf{e}_1, & [\mathbf{e}_2,\mathbf{e}_4]=\mathbf{e}_6,\\
[\mathbf{e}_2,\mathbf{e}_5]=\mathbf{e}_7, & [\mathbf{e}_2,\mathbf{e}_6]=-\mathbf{e}_4, & [\mathbf{e}_2,\mathbf{e}_7]=-\mathbf{e}_5, & [\mathbf{e}_3,\mathbf{e}_4]=\mathbf{e}_7,\\
[\mathbf{e}_3,\mathbf{e}_5]=-\mathbf{e}_6, & [\mathbf{e}_3,\mathbf{e}_6]=\mathbf{e}_5, & [\mathbf{e}_3,\mathbf{e}_7]=-\mathbf{e}_4, & [\mathbf{e}_4,\mathbf{e}_5]=\mathbf{e}_1,\\
[\mathbf{e}_4,\mathbf{e}_6]=\mathbf{e}_2, & [\mathbf{e}_4,\mathbf{e}_7]=\mathbf{e}_3, & [\mathbf{e}_5,\mathbf{e}_6]=-\mathbf{e}_3, & [\mathbf{e}_5,\mathbf{e}_7]=\mathbf{e}_2,\\
[\mathbf{e}_6,\mathbf{e}_7]=-\mathbf{e}_1, & [\mathbf{e}_i,\mathbf{e}_i]=0, & i=1,\dots,7. &
\end{array}$$

The manifolds with the structure group $G_2$ were studied in [2], [3], etc. Note that the group $COG_2$ is a conformal analog of the automorphism group $G_2$ of the octave algebra, i.e., $COG_2 = \{\lambda\cdot A \mid \lambda \in R^+,\ A \in G_2\}$. In the seven-dimensional space the octave geometry is the most appropriate geometry for investigation because for small structure groups the tangent spaces of manifolds and submanifolds are extremely non-isotropic. At the same time, if the structure group is large (e.g., $SL(7)$), the number of invariants is small. If the dimension is arbitrary, from this point of view the orthogonal and conformal groups are the most appropriate structure groups. And for the seven-dimensional space the most appropriate structure group is the compact form and the normal non-compact form of the Lie group $G_2$. This group is also convenient for computer calculations, since the other exceptional groups (e.g., 52-dimensional $F_4$ with the standard 26-dimensional representation space) require considerably more long and complicated calculations.

The differential geometry of seven-dimensional manifolds and submanifolds in the spaces with structure group $G_2^n$ (the normal form of the exceptional complex Lie group $G_2^c$) is very rich in properties as compared even to the standard octave geometry.

The geometric objects with the structure group $COG_2$ excel in peculiarity of their properties and are of significant interest not only for mathematical investigation, but for application in physics as well [4]. Hence it is quite natural to study spaces with the fundamental group $G_2$ as well as the spaces with the fundamental groups $G_2^n$ and $COG_2$. The space with fundamental group $G_2^n$ will be called the pseudooctave space, and the space with the fundamental group $COG_2$ will be called the conformal-octave space.

In the present paper we consider immersed smooth hypersurfaces with non-isotropic normals in these spaces and prove analogs of the Bonnet theorem for these hypersurfaces.