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THE ONSAGER EQUATION AS THE LYAPUNOV–SCHMIDT EQUATION

L.D. Eskin

1. L. Onsager (see, e.g., [1], p. 50) investigated in detail thermodynamic properties of a system of strongly extended rigid (non-deformable) cylindric rods ($\delta = dl^{-1} \ll 1$, d is the diameter, l is the length of a rod) with pair interaction of the type of a steric repulsion (the Onsager model of excluded volume). It was shown that, in a system which is orientation-disordered for low concentrations, along with the growth of concentration a first genus phase transition into an anisotropic (orientation-ordered) phase occurs, which was treated as a liquid crystal nematic. All thermodynamic properties of the isotropic phase are described by a uniform function of distributions of particle axes orientations with the density $f(n) = 1$ (n is the unit vector of the rod's axis). The thermodynamic properties of the nematic are described by a density f which differs from unit and has the unique maximum in the director's direction (the direction of prevailing orientation of axes), which is invariant with respect to both rotations around this direction and the change $n \to -n$. For $f(n)$ from the condition of the minimum of free energy of rods system, which was found in the approximation of the second virial coefficient, Onsager obtained the nonlinear integral equation

$$\nu + \ln f(n') + \lambda \int B(n, n') f(n) d^2 n = 0. \tag{1.1}$$

In (1.1), $\lambda = 2cdl^2$ (c is the system density) is a dimensionless parameter, the kernel is

$$B = (1 - (nn')^2)^{1/2},$$

nn' stands for the scalar product of unit vectors n and n', unknown constant ν is determined by the norming condition for $f(n)$

$$\int f(n) d^2 n = 1, \tag{1.2}$$

$d^2 n$ is a sphere surface element, it is determined in spherical coordinate system with the polar axes in director's direction by means of the relation

$$d^2 n = \frac{1}{4\pi} \sin\theta d\varphi\, d\theta$$

(φ, θ are spherical coordinates of the unit vector n). Since the density describes the nematic, it is a solution of integral equation (1.1), which, in addition to the norming condition (1.2), satisfies also the following conditions:

a) $f(n)$ does not depend on the angle φ ($f(n) = f(\theta)$),

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