ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2010, Vol. 54, No. 4, pp. 1–6. © Allerton Press, Inc., 2010.
Original Russian Text © I.I.Batyrshin, 2010, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2010, No. 4, pp. 3–9.

Isolated 2-Computably Enumerable Q-Degrees

I.I. Batyrshin[1*]

[1] *Postgraduate, Kazan State University, 18 Kremlevskaya str., Kazan, 420008 Russia*

Received March 3, 2008

Abstract—In this work we prove that for every pair of computably enumerable degrees $\mathbf{a} <_{\mathbf{Q}} \mathbf{b}$ there exists a properly 2-computably enumerable degree $\mathbf{d}$ such that $\mathbf{a} <_{\mathbf{Q}} \mathbf{d} <_{\mathbf{Q}} \mathbf{b}$, $\mathbf{a}$ isolates $\mathbf{d}$ from below, and $\mathbf{b}$ isolates $\mathbf{d}$ from above. Two corollaries follow from this result. First, there exists a 2-computably enumerable degree which is Q-incomparable with any nontrivial (different from $\mathbf{0}$ and $\mathbf{0}'$) computably enumerable degree. Second, every nontrivial computably enumerable degree isolates some 2-computably enumerable degree from below and some 2-computably enumerable degree from above.

DOI: 10.3103/S1066369X10040018

Key words and phrases: *computably enumerable sets, quasi-reducibility, 2-computably enumerable sets, isolated degrees.*

1. INTRODUCTION

The notion of quasi-reducibility was first defined by Tennenbaum ([1], p. 207) as a particular case of a reducibility different from the T-reducibility on the class of computably enumerable (c. e.) sets. In accordance with this definition, a set A is quasi-reducible to a set B ($A \leq_Q B$) if there exists a computable function g such that for any $x \in \omega$,

$$x \in A \Leftrightarrow W_{g(x)} \subseteq B.$$

Later on, a study of combinatorial properties of various sets with the help of the quasi-reducibility (the Q-reducibility) led to several interesting results. The most known result among them is the Marchenkov solution [2] of the well-known Post problem on the existence of an incomplete noncomputable c. e. degree.

The relation $A \leq_Q B$ is reflexive and transitive, therefore it generates the equivalence relation that forms the structure of quasi-degrees (Q-degrees) on 2^ω.

The interest to studying the algebraic structure of Q-degrees was arisen due to the works of Dobritsa and Belegradek [3]. The results obtained by them imply that information on properties of this structure can be helpful in clarifying properties of classes of finitely generated subgroups of algebraically closed groups.

Arslanov and Omanadze were first to study the algebraic structure of Q-degrees of n-c. e. sets [4]. Here we continue their research and, in particular, generalize some theorems from [4].

This paper is dedicated to studying isolated quasi-degrees of 2-c. e. sets. A definition of isolated Turing degrees of 2-c. e. sets was introduced by Cooper and Yi. The study of properties of such degrees is one of the main directions in the study of the algebraic structure of T-degrees.

In this paper we use the usual denotations (see, e.g., [5]). In particular, the symbol $[s]$ after the sign of a functional and in formulas means that the value of the functional or a parameter in a formula is calculated at stage s. Small lower-case roman letters denote quasi-degrees. In the paper we use the following definitions.

[*] E-mail: batyrshin@gmail.com.