Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 46, No. 5, pp. 59–67, 2002 *UDC 512*

OPERADS IN THE CATEGORY OF CONVEXORS. II

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This is the second part of the article started in [1]. The results of both the parts were announced in [2]. In the second part we introduce a convexor analog for a linear operad, or a conoperad, i. e., an operad whose components are convexors and whose operations of composition are multilinear in the sense of the theory of convexors. Examples of operads of such kind are the operads of multidimensional stochastic and doubly stochastic matrices, whose first components are sets of square stochastic and doubly stochastic matrices, respectively. Apparently, only in the recent times the researchers working in the theory of probability started to seek the approaches to the study of these objects (see, e. g., [3]). After an appropriate reformulation, probabilistic automata become elements of the algebras over multidimensional stochastic matrices. As for the main result of this article, it is the characterization of varieties of conalgebras over conoperads: these are exactly the varieties determined by convexor analogs of multilinear identities. This is analogous to the situation in the linear case (see [4]–[7]).

By a Δ-linear operad an operad $\mathfrak{R}$ is called such that the correspondence $n \mapsto \mathfrak{R}(n)$ is a functor to Conv and the mappings of composition are Δ-multilinear. Following [8], [9], we may call this operad a conoperad. By an algebra over a Δ-linear operad $\mathfrak{R}$ (or, following [8], [9], an $\mathfrak{R}$-conalgebra) we will call an $\mathfrak{R}$-algebra A (which is a convexor) in which all operations of composition $\mathfrak{R}(n) \times A^n \to A$ are Δ-multilinear. Homomorphisms between such algebras are assumed to be Δ-linear. Up to the end of the article Alg-$\mathfrak{R}$ will stand for the category of algebras over a Δ-linear operad. The category of algebras and homomorphisms Alg-$\mathfrak{R}$ is a variety of Δ-linear multioperator algebras in the sense of [1]. The proper operad $\mathfrak{R}$ is treated as the signature; moreover, for all $n \geq 1$, $\mathfrak{R}(n)$ is a set of symbols of n-ary Δ-multilinear operations.

The simplest example of a Δ-linear operad is the operad $\mathfrak{C}$ in which the set $\mathfrak{C}(n)$ is one-element for each n, $\mathfrak{C}(n) = \{\omega_n\}$. Such a set possesses the intrinsic structure of a convexor: $\alpha(\omega_n, \omega_n) = \omega_n$. The operation of composition is defined in the unique possible way: $\omega_m \omega_{n_1} \dots \omega_{n_m} = \omega_{n_1+\dots+n_m}$. Clearly, these mappings are Δ-multilinear. For any subcategory **K** of the category of finite ordinals **FSet**, satisfying the conditions formulated in [1], on $\mathfrak{C}$ the structure of **K**-operad can be defined in a trivial way.

Recall that two varieties of algebras $\mathfrak{V}_1$ and $\mathfrak{V}_2$ are said to be rationally equivalent if 1) they are equivalent as the categories 2) while the equivalence functor assigns to an algebra from $\mathfrak{V}_1$, which is a set A with a certain family of operations, an algebra from $\mathfrak{V}_2$, which is the same set A but with possibly another set of operations. As mappings between sets, homomorphisms from $\mathfrak{V}_1$ pass to homomorphisms from $\mathfrak{V}_2$, which are the same mappings. The inverse functor acts in just the same manner. Thus, by their essence, the algebras from the two varieties are the same sets, and the operations defining the structures of the algebras of the two types can be expressed via each other (are mutually derivative). The rational equivalence of varieties is equivalent to the

Supported partially by the Russian Foundation for Basic Research (grant no. 99-01-00469).