Russian Mathematics (Iz. VUZ)
Vol. 42, No. 11, pp.61–64, 1998

Izvestiya VUZ. Matematika
UDC 512.542, 512.531.2

REPRESENTATION OF SEMIGROUP OF ENDOMORPHISMS OF A FINITE GROUP ON THE SPACE OF ITS CLASS FUNCTIONS

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Clearly, the structure of a group and the structure of the semigroup of its endomorphisms are things closely interconnected. Works of a series of authors were dedicated to the study of this interconnection (see, e.g., [1], [2]). It turned out that, in the case of finiteness of the group G, a close connection between the semigroup of endomorphisms of this group and the linear space of class functions from G to the field of complex numbers can be established.

All groups under consideration are finite, and linear spaces over them are given over the field of complex numbers. By *representation* we shall mean a matrix representation over the field of complex numbers. In this case, for a group the correspondence between its representation and the character of such a representation is one-to-one (up to an isomorphism). So, we shall speak on "the kernel of a character", "irreducible character", and so on. Let G be a group, then $\operatorname{Irr} G$ is the set of all its irreducible characters; besides, if necessary, we shall assume it to be linearly ordered. A function with the range G and values in the field $\mathbf{C}$ of complex numbers will be called a *class function* if its values are same on any elements conjugated in the group G. We denote by V_G a linear space of class functions of the group G. As known, $\operatorname{Irr} G$ is a basis for V_G (see [3]). If φ is a mapping from the set A to the set B, then we denote by x^φ the image of an element x from A under such mapping; M^φ will stand for the image of a subset M from A. If φ is a homomorphism of the group G onto H, and χ is the character of a certain representation ψ of the group G, which contains the kernel φ in its kernel, then we denote by χ^φ the character of the representation ψ^φ, "induced" by the representation ψ on $G^\varphi = H$ (as homomorphic image of the group G). But if the kernel of the character χ does not contain the kernel of the homomorphism φ, then we put χ^φ equal to the zero vector $\mathbf{O}$ of the space V_G. If $\mathbf{O}$ is the zero vector from V_G, then we put $\mathbf{O}^\varphi$ equal to the zero vector of the space V_H. If S is subgroup of the group G and η is the character of representation θ of group S, then η^G is the character of the representation θ^G induced from S to G. A matrix is called *matrix of permutation* (permutational matrix) if in both every its column and every its line all elements are zero except one, equaling 1. If X is a matrix, then X^t is the matrix transposed to X.

To each endomorphism φ of the group G we put into correspondence in a way described below a linear operator $\mathbf{A}_\varphi$ (respectively, $\mathbf{B}_\varphi$) mapping the space V_G into itself. The main objective of the present article is the proof of Theorem 1, which ascertains that such a correspondence is a representation (an antirepresentation, respectively).

It is known that for a univalent definition of the operators $\mathbf{A}_\varphi$ and $\mathbf{B}_\varphi$ it suffices to define the action of the operators $\mathbf{A}_\varphi$ and $\mathbf{B}_\varphi$ on a basis on the space V_G, for example, on $\operatorname{Irr} G$ (see [3], lemma 11). Let $\operatorname{Irr} G = \{\chi_1, \chi_2, \dots, \chi_r\}$. If an irreducible character χ_i contains in its kernel the kernel of the endomorphism φ, then we set

$$\chi_i^{\mathbf{A}_\varphi} = (\chi_i^\varphi)^G = \sum_{j=1}^{r} a_{ij}^{(\varphi)} \chi_j.$$

Supported by Russian Foundation for Basic Research (code 94-01-00802-a).