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T-Irreducible Extension of Polygonal Digraphs

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Abstract—Directed graphs are mathematical models of discrete systems. T-irreducible extensions are widely used in cryptography and diagnosis of discrete systems. A polygonal origraph is a digraph obtained from a circuit by some orientation of its edges. We propose an algorithm to construct a T-irreducible extension of a polygonal graph.

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INTRODUCTION

We say that a *directed graph* (*digraph*) is a pair $\overrightarrow{G} = (V, \alpha)$ consisting of nonempty digraph vertex set V and a digraph edge defining relation α on the set V. A digraph edge is *incident* to the vertex v if v is either the starting or ending point of this edge. An *inclusion* of digraph $\overrightarrow{G} = (V, \alpha)$ into the digraph $\overrightarrow{H} = (W, \beta)$ is a bijective map $\phi : V \to W$ such that $((u, v) \in \alpha \to (\phi(u), \phi(v)) \in \beta) \quad \forall u, v \in V$. In this case we say that the digraph $\overrightarrow{G}$ can be included into the digraph $\overrightarrow{H}$. A *part* of digraph $\overrightarrow{G} = (V, \alpha)$ is a digraph $\overrightarrow{H} = (W, \beta)$ such that $W \subseteq V$ and $\beta \subseteq (W \times W) \cap \alpha$. A part of digraph $\overrightarrow{G} = (V, \alpha)$ is a *subgraph* of digraph $\overrightarrow{H} = (W, \beta)$ if $\beta = (W \times W) \cap \alpha$. A subgraph $\overrightarrow{H}$ is *maximal* if it coincides with the initial digraph $\overrightarrow{G}$ without one vertex v and all the edges incident to it. An *extension* of digraph $\overrightarrow{G} = (V, \alpha)$ is the digraph $\overrightarrow{H} = (W, \beta)$ such that, firstly, $|W| = |V| + 1$ and, secondly, the digraph $\overrightarrow{G}$ is includable into each maximal subgraph of the digraph $\overrightarrow{H}$. Consider digraphs $\overrightarrow{G} = (V, \alpha)$ and $\overrightarrow{H} = (W, \beta)$ such that $V \cap W = \varnothing$. Their *union* is the digraph $\overrightarrow{G} \cup \overrightarrow{H} = (V \cup W, \alpha \cup \beta)$. Now for digraphs $\overrightarrow{G} = (V, \alpha)$ and $\overrightarrow{H} = (W, \beta)$ such that $V \cap W = \varnothing$ we introduce the notion of a *composition* as the digraph $\overrightarrow{G} + \overrightarrow{H} = (V \cup W, \alpha \cup \beta \cup V \times W \cup W \times V)$. An *isomorphism* of digraphs $\overrightarrow{G} = (V, \alpha)$ and $\overrightarrow{H} = (W, \beta)$ is a bijective correspondence $\phi : V \to W$ preserving the adjacency relation, i.e., $((u, v) \in \alpha \leftrightarrow (\phi(u), \phi(v)) \in \beta) \quad \forall u, v \in V$. We denote the *isomorphism relation* on digraphs $\overrightarrow{G}$ and $\overrightarrow{H}$ by $\overrightarrow{G} \cong \overrightarrow{H}$. The digraphs $\overrightarrow{G}$ and $\overrightarrow{H}$ are then said to be *isomorphic* [1].

A *trivial extension* (TE) of the digraph $\overrightarrow{G} = (V, \alpha)$ is a composition $\overrightarrow{G} + w$ of the initial digraph $\overrightarrow{G}$ with some vertex $w \notin V$. Since the digraph $\overrightarrow{G}$ trivial extension is unique up to some isomorphism, it is possible to introduce the function TE($\overrightarrow{G}$). A *T-irreducible extension* (TIE) of the digraph $\overrightarrow{G}$ is the extension of the initial graph $\overrightarrow{G}$ being the result of the maximal possible number of edges elimination from TE($\overrightarrow{G}$) [2]. S. G. Kurnosova considered TIE for some cases of undirected (with symmetrical or anti-reflexive equivalence relation) graphs in [2, 3]. The authors of [3] endowed complete binary trees with TIE.

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