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# Depression Conditions for Linear Equations with Major Partial Derivatives

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We consider equations in the form

$$u_{(m_1,\dots,m_n)}(x)+\Big[\sum_{i_1=0}^{m_1}\cdots\sum_{i_n=0}^{m_n}\Big]a^{(i_1,\dots,i_n)}(x)u_{(i_1,\dots,i_n)}(x)=f(x), \tag{1}$$

where $x=(x_1,\dots,x_n)$ are points of a certain domain $D$ of a Euclidean space, $m_k$, $i_k$, $k=\overline{1,n}$, are integer nonnegative numbers, $u_{(i_1,\dots,i_n)}(x)=\frac{\partial^{i_1+\cdots+i_n}u(x)}{\partial x_1{}^{i_1}\dots\partial x_n{}^{i_n}}$, $\Big[\sum\limits_{i_1=0}^{m_1}\cdots\sum\limits_{i_n=0}^{m_n}\Big]=\sum\limits_{i_1+\cdots+i_n\le r-1}$, $r=m_1+\cdots+m_n$.

Eqs. (1) arise in various applications, namely, the filtration of fluids in porous media [1], the absorption of soil waters by roots of plants ([2], p. 262), the integral representations of transforms of differential operators ([3], pp. 5–13), the theory of approximation and mappings ([4], pp. 63, 109), et al. Several results are surveyed in monograph [5]. In this paper, we formulate the conditions for the coefficients in (1), ensuring the depression of the equation. We omit the argument $x$ in the equation and use the notation

$$(x_1,\dots,t,\dots,x_n)=(x_1,\dots,x_{j-1},t,x_{j+1},\dots,x_n),$$
$$(i_1,\dots,k,\dots,i_n)=(i_1,\dots,i_{j-1},k,i_{j+1},\dots,i_n),$$
$$\Big[\sum_{i_1=0}^{m_1}\cdots\sum_{i_j=0}^{k}\cdots\sum_{i_n=0}^{m_n}\Big]=\Big[\sum_{i_1=0}^{m_1}\cdots\sum_{i_{j-1}=0}^{m_{j-1}}\sum_{i_j=0}^{k}\sum_{i_{j+1}=0}^{m_{j+1}}\cdots\sum_{i_n=0}^{m_n}\Big],$$
$$a^{(i_1,\dots,i_{j-1},k,i_{j+1},\dots,i_n)}=a^{(i_1,\dots,k,\dots,i_n)},\quad u_{(i_1,\dots,i_{j-1},k,i_{j+1},\dots,i_n)}=u_{(i_1,\dots,k,\dots,i_n)}.$$

**1.** Let us first formulate the conditions which enable us to represent (1) as a system of two equations such that in one of them the desired function is differentiated only in one independent variable.

**Theorem 1.** *Assume that with a certain $j$ $(1\le j\le n)$ such that $m_j>1$ the coefficients of the equation satisfy the conditions $\frac{\partial^{i_j}a^{(i_1,\dots,i_n)}}{\partial x_j^{i_j}}$, $f\in C(D)$, $i_k=\overline{0,m_k}$, $k=\overline{1,n}$. If the following identities are true*:

$$a^{(i_1,\dots,i_n)}-C_{m_j}^{i_j}\frac{\partial^{m_j-i_j}}{\partial x_j^{m_j-i_j}}a^{(i_1,\dots,m_j,\dots,i_n)}-\sum_{i=i_j}^{m_j-1}a^{(m_1,\dots,i,\dots,m_n)}C_i^{i_j}\frac{\partial^{i-i_j}}{\partial x_j^{i-i_j}}a^{(i_1,\dots,m_j,\dots,i_n)}\equiv 0, \tag{2}$$

*$i_k=\overline{0,m_k}$, $k\ne j$, $i_j=\overline{0,m_j-1}$, $i_1+\cdots+i_{j-1}+i_{j+1}+\cdots+i_n<r-m_j$, then Eq.* (1) *takes the form*

$$u_{(m_1,\dots,0,\dots,m_n)}+\Big[\sum_{i_1=0}^{m_1}\cdots\sum_{i_{j-1}=0}^{m_{j-1}}\sum_{i_{j+1}=0}^{m_{j+1}}\cdots\sum_{i_n=0}^{m_n}\Big]a^{(i_1,\dots,m_j,\dots,i_n)}u_{(i_1,\dots,0,\dots,i_n)}=v;$$

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