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One Sufficient Condition for Hamiltonian Graphs Involving Distances

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Abstract—In 1990 G. T. Chen proved that if G is a 2-connected graph of order n and $2|N(x) \cup N(y)| + d(x) + d(y) \geq 2n - 1$ for each pair of nonadjacent vertices $x, y \in V(G)$, then G is Hamiltonian. In this paper we prove that if G is a 2-connected graph of order n and $2|N(x) \cup N(y)| + d(x) + d(y) \geq 2n - 1$ for each pair of nonadjacent vertices $x, y \in V(G)$ such that $d(x, y) = 2$, then G is Hamiltonian.

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1. INTRODUCTION

In this paper we consider finite undirected graphs without loops and multiple edges. For a graph G we denote by $V(G)$ and $E(G)$ the sets of its vertices and edges, respectively; the symbol $\delta(G)$ stands for the minimum degree of G. For subsets H and S of the set $V(G)$ (representing subgraphs of G) we denote by $N_H(S)$ the set of vertices that belong to H and are adjacent to some vertex in S. In particular, if $H = G$ and $S = \{u\}$, then $N_G(S) = N(u)$. We denote by $G - H$ and $G[S]$ subgraphs of G induced on $V(G) - V(H)$ and S, respectively. Let $C_m = x_1x_2 \cdots x_m x_1$ be a cycle of order m. Define

$$N^+_{C_m}(u) = \{x_{i+1} : x_i \in N_{C_m}(u)\}, \quad N^-_{C_m}(u) = \{x_{i-1} : x_i \in N_{C_m}(u)\},$$

and $N^{\pm}_{C_m}(u) = N^+_{C_m}(u) \cup N^-_{C_m}(u)$, where subscripts are taken modulo m.

In 1960 Ø. Ore imposed the following condition on the sum of degrees of vertices of a graph guaranteeing its Hamiltonian property.

Theorem 1 ([1]). *If G is a connected graph of order $n \geq 3$ and $d(x) + d(y) \geq n$ for each pair of its nonadjacent vertices x, y, then the graph G is Hamiltonian.*

In 1989 R. J. Faudree et al. introduced a neighborhood union condition guaranteeing the Hamiltonian property of a graph.

Theorem 2 ([2]). *If G is a 2-connected graph of order $n \geq 3$ and $|N(x) \cup N(y)| \geq (2n - 1)/3$ for each pair of its nonadjacent vertices x, y, then the graph G is Hamiltonian.*

In 1991 D. Bauer et al. improved Theorem 2 as follows.

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