ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2011, Vol. 55, No. 8, pp. 38–46. © Allerton Press, Inc., 2011.
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Finite Groups, whose Primary Subgroups are either F-Subnormal or F-Abnormal

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Received April 16, 2010

Abstract—We study finite groups whose each primary subgroup is either subnormal or abnormal with respect to classes of all nilpotent, all p-closed, and all p-nilpotent groups. In particular, we fully describe these groups.

DOI: 10.3103/S1066369X1108007X

Keywords and phrases: *primary groups, subnormality, abnormality, p-nilpotent groups.*

We consider only finite groups. In the paper [1] one has described finite groups, whose any proper subgroup is either subnormal or abnormal. In the theory of classes of finite groups a natural generalization of the subnormality and abnormality is the notion of $\mathfrak{F}$-subnormality and $\mathfrak{F}$-abnormality.

Let $\mathfrak{F}$ be a nonempty formation. A subgroup K of a group G is called $\mathfrak{F}$-subnormal if either $K = G$ or there exists a maximal chain

$$G = K_0 \supset K_1 \supset \cdots \supset K_n = K$$

such that $(K_{i-1})^{\mathfrak{F}} \subseteq K_i$ for all $i = 1, 2, \ldots, n$.

A subgroup H of a group G is called $\mathfrak{F}$-abnormal, if either $H = G$ or any maximal chain

$$G = H_0 \supset H_1 \supset \cdots \supset H_n = H$$

is such that $H_i(H_{i-1})^{\mathfrak{F}} = H_{i-1}$ for all $i = 1, 2, \ldots, n$.

In the paper [2] one has studied the structure of finite groups, whose any proper subgroup is either $\mathfrak{F}$-subnormal or $\mathfrak{F}$-abnormal for a local formation $\mathfrak{F}$. In this paper we consider the structure of finite groups, whose any primary subgroup is either $\mathfrak{F}$-subnormal or $\mathfrak{F}$-abnormal. In particular, we fully describe such groups when $\mathfrak{F}$ is the class of all nilpotent, all p-nilpotent, or all p-closed groups.

Recall that a group, whose order is a degree of a prime number, is said to be primary. A formation $\mathfrak{F}$ is a class of groups closed with respect to homomorphic images and subdirect products. A local formation is a formation closed with respect to Frattini extensions. The subgroup $G^{\mathfrak{F}}$ is the intersection of all subgroups N of the group G, for which G/N belongs to $\mathfrak{F}$. We denote by $\pi(\mathfrak{F})$ the characteristic of the formation $\mathfrak{F}$, i.e., the set of all primes that divide orders of groups from $\mathfrak{F}$.

See [3] for all other definitions and denotations.

Lemma 1. *Let $\mathfrak{F}$ be a local hereditary formation. If all primary subgroups in a group G are $\mathfrak{F}$-subnormal, then G satisfies one of the following conditions*:

1) $G \in \mathfrak{F}$;

2) $|G| = p$, *where* $p \notin \pi(\mathfrak{F})$;

3) $\left|G : G^{\mathfrak{F}}\right|$ *is divisible by at least by two different primes.*

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