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THE BOUNDED ON THE AXIS SOLUTIONS OF LINEAR EQUATIONS OF THE SOBOLEV TYPE WITH RELATIVELY SECTORIAL OPERATORS

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Introduction

Assume that $\mathcal{U}$ and $\mathcal{F}$ are Banach spaces; $L \in \mathcal{L}(\mathcal{U};\mathcal{F})$ is a linear continuous operator, $\ker L \neq \{0\}$; $M \in \mathcal{C}l(\mathcal{U};\mathcal{F})$ is a linear closed operator with a dense definition domain; $f : \mathbb{R} \to \mathcal{F}$ is a certain function. Many initially boundary value problems for equations and systems of equations which model various real-world processes ([1]; [2], pp. 13–15) are reduced to the Cauchy problem

$$u(0) = u_0 \tag{1}$$

for an operator-differential equation of the Sobolev type [1]–[4]

$$L\dot{u}(t) = Mu(t) + f(t). \tag{2}$$

In this paper, we study bounded at the whole axis $\mathbb{R}$ classical solutions of equation (2) or the corresponding Cauchy problem.

If the operator $L^{-1} \in \mathcal{L}(\mathcal{F};\mathcal{U})$ exists, one can reduce equation (2) to that $\dot{u}(t) = Su(t) + w(t)$, where $S = L^{-1}M \in \mathcal{C}l(\mathcal{U})$, $\operatorname{dom} S = \operatorname{dom} M$, $w(t) = L^{-1}f(t) : \mathbb{R} \to \mathcal{U}$. Conditions which ensure the existence of bounded solutions of equations of such a type with the operator $S \in \mathcal{L}(\mathcal{U})$ are obtained in ([5], pp. 118–131).

In [6]–[8], the similar questions are studied for the nonstationary equation

$$\dot{u}(t) = S(t)u(t) + w(t), \quad t \in J \subset \mathbb{R}.$$

In addition, in ([6], pp. 145–343), the operator-function $S(t) : J \to \mathcal{L}(\mathcal{U})$ is investigated. The more general case is considered in ([7], pp. 165–181; [8], pp. 245–252), where the values of the operator-function $S(t)$ are unbounded operators for $t \in J$. In addition, in ([6], pp. 145–343; [7], pp. 165–181), the conditions of the existence of bounded solutions and those from the more general classes are obtained.

The questions about the existence of bounded solutions of equation (2) are studied in [9], where this equation is considered with a (L,σ)-bounded operator M and with a strongly (L,p)-sectorial one. In the latter case, only solutions defined on the positive semi-axis are taken into account.

In order to investigate the questions about the existence of bounded on the entire axis solutions of equation (2) with a strongly (L,p)-sectorial unbounded operator M, we apply the methods used in ([5], pp. 118–131 and [9]). In this paper, we obtain sufficient conditions for the existence of a unique bounded on the entire axis solution of equation (2) and the Cauchy problem for it. In

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