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The Structure of the Resolvent for the Discrete Renewal Equation with Nonsummable Difference Kernel

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Abstract—We find the asymptotic structure of the resolvent for the various cases of zeros of symbol for the discrete difference renewal equations with the nonsummable kernel.

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Let us consider the equation

$$x_n = \sum_{k=0}^{n} a_{n-k}x_k + f_n, \quad n = 0, 1, 2, \dots, \tag{1}$$

where $a_0 \neq 1$. For sequences $a = \{a_n\}$ we define the generating function $\widehat{a}(z) = \sum\limits_{n=0}^{\infty} a_n z^n$ and the convolution $a * b = \Big\{\sum\limits_{k=0}^{n} a_{n-k}b_k\Big\}$. We denote by $\delta = (1, 0, \dots, 0, \dots)$ a unit sequence, i.e., $\delta * a = a$. The solution to equation $(\delta - a) * x = f$ can be presented in the form $x = (\delta + r) * f$, where r is the resolvent of kernel a, i.e., the sequence satisfying the equation $r = (\delta + r) * a$. We also introduce a function of natural argument n in the form $\psi^{(\beta)}(\lambda) = \{\psi(n, \beta)\lambda^n\}$, $\lambda \in \mathbb{C}$, $\beta \in \mathbb{R}$, where $\psi(n, \beta) = \frac{\prod_{i=1}^{n}(\beta+i)}{n!}$. The sequence $\psi^{(\beta)}(\lambda)$ decreases monotonically with $\beta < 0$ and $|\lambda| \leq 1$.

In addition, we denote by l_1 the space of sequences a such that $\sum\limits_{n=0}^{\infty} |a_n| < \infty$, and by $P_r(n)$ a polynomial of degree no more than r. We set $\prod\limits_{i=1}^{0} d_i = 1$. We note that any polynomial $P_k(n)$ can be presented in the form

$$P_k(n) = \sum_{r=0}^{k} c_r \psi(n, r). \tag{2}$$

For sequences $a \in l_1$ the structure of resolvent $r^{(a)}$ of kernel a, and hence, the asymptotic of solution (1) are investigated rather well (see, e.g., [1, 2]).

In the case $a \notin l_1$ the asymptotic structure of resolvent has been found for kernel in the form

$$a_n = \sum_{j=1}^{p} P_{n_j}(n)\mu_j^{-n} + a_n^{(0)},$$

where $|\mu_j| \leq 1$, $a^{(0)} \in l_1$ (by theorem 2 from [3]). With that one has used the idea of papers [1, 4]. With the help of replacement $x = (\delta - b) * y$ Eq. (1) is reduced to the equation with the kernel $c \in l_1$

$$\delta - c = (\delta - a) * (\delta - b) \tag{3}$$

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