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MULTIPLICATIVE PROPERTIES OF BLASCHKE PRODUCTS IN SOME CLASSES OF FUNCTIONS HOLOMORPHIC IN A DISK

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1. Let $D = \{z \in \mathbb{C}, |z| < 1\}$ be the unit disk, $T = \{z \in \mathbb{C}, |z| = 1\}$ be its boundary, $H(D)$ be the set of all holomorphic in D functions, $m_2(z)$ and $m(t)$ stand for normed Lebesgue measures on the disk D and on its boundary T, respectively, X and Y be subspaces in $H(D)$. Further, let $\{z_k\}_{k=1}^{\infty}$, $0 < |z_k| < 1$, be an arbitrary sequence of complex numbers, such that $|z_k| < |z_{k+1}|$, $k \in N$, $B(z, \{z_k\})$ be the corresponding Blaschke product (see [1]).

Definition. We will say that the Blaschke product $B(z, \{z_k\}) = B(z)$ generates a bounded operator

$$T_B : (T_B f)(z) = \int_T f(rt)B(\bar{t}w)dm(t), \quad z = rw,$$

which acts from X to Y (or is a multiplier from X to Y) if, for any function $f \in X$, the function $T_B f$ lies in Y.

In the case where Q is an arbitrary function holomorphic in disk (or polydisk) D, the complete description of Q for which the operator T_Q acts boundedly from X to Y, where X and Y are distinct pairs of spaces (Hardy classes H^p, $0 < p < 1$, spaces with mixed norm) holomorphic in a disk (polydisk), was given in [2]–[4].

The objective of this article is to determine necessary and sufficient conditions upon $B(z, \{z_k\})$, in particular, in terms of sequences of zeros $\{z_k\}_{k=1}^{\infty}$ of the function $B(z, \{z_k\})$, under which the operator T_B is a bounded operator from X to Y.

Let us note that the relations between the Taylor coefficients, the growth of mean-values $M_p(B^{(m)}, r)$, where

$$M_p(f, r) = \int_T |f(rt)|^p dm(t), \quad r \in (0,1), \quad f \in H(D), \quad 0 < p < \infty,$$

and the distribution of zeros $\{z_k\}$ of the Blaschke products were investigated by many authors (see, e.g., [5], [6]).

2. To state the main results of this article we give the following

Definition (see [1]). We say that

1) the Blaschke product $B(z, \{z_k\})$ with zeros $\{z_k\}_{k=1}^{\infty}$ is interpolation one (generalized interpolation, respectively) if

$$\delta(B) = \inf_j |B'(z_j)|(1 - |z_j|) > 0 \quad (\delta_\gamma(B) = \inf_j |B'(z_j)|(1 - |z_j|)^\gamma > 0, \quad \gamma > 0, \quad \text{respectively});$$

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