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A Nonlinear Boundary Eigenvalue Problem for TM-Polarized Electromagnetic Waves in a Nonlinear Layer

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Abstract—We consider the propagation of TM-polarized electromagnetic waves in a nonlinear dielectric layer located between two linear media. The nonlinearity in the layer is described by the Kerr law. We reduce the problem to a nonlinear boundary eigenvalue problem for a system of ordinary differential equations. We obtain a dispersion relation and a first approximation for eigenvalues of the problem. We compare the results with those obtained for the case of a linear medium in the layer.

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Key words and phrases: *dispersion relation, boundary-value problem, Kerr nonlinearity.*

Brief communication

INTRODUCTION

The problems of the propagation of nonlinear electromagnetic waves in nonlinear media have been studied intensively for several decades [1—5]. Mathematical models for such phenomena lead to boundary eigenvalue problems for a system of ordinary differential equations which are nonlinear with respect to the spectral parameter. Such problems are difficult to study, because the known methods for spectral problems appear to be inefficient. In spite of a large number of publications, the complete analytic solution to the problem of the propagation of TM-polarized waves in a nonlinear dielectric layer is not obtained yet.

1. PROBLEM DEFINITION

Consider electromagnetic waves which propagate through a homogeneous isotropic nonmagnetic dielectric layer with the Kerr-type nonlinearity. In the Cartesian system of coordinates $Oxyz$ the layer is located between two semi-infinite media $x < 0$ and $x > h$. The media are filled with an isotropic nonmagnetic medium without sources and have constant dielectric permittivities $\varepsilon_1 \geq \varepsilon_0$ and $\varepsilon_3 \geq \varepsilon_0$, correspondingly, where ε_0 is the dielectric permittivity of the vacuum. We assume that this value everywhere is $\mu = \mu_0$.

The electric field $\mathbf{E}(x, y, z, t) = \mathbf{E}_+(x, y, z)\cos\omega t + \mathbf{E}_-(x, y, z)\sin\omega t$ harmonically depends on time and satisfies Maxwell's equations

$$\operatorname{rot}\mathbf{H} = -i\omega\varepsilon\mathbf{E}, \ \ \operatorname{rot}\mathbf{E} = i\omega\mu\mathbf{H}, \tag{1.1}$$

where $\mathbf{E}(x, y, z) = \mathbf{E}_+(x, y, z) + i\mathbf{E}_-(x, y, z)$ and $\mathbf{H}(x, y, z)$ are complex amplitudes. Inside the layer the dielectric permittivity obeys the Kerr law $\varepsilon = \varepsilon_2 + a|E|^2$, where $a > 0$ and $\varepsilon_2 > \max(\varepsilon_1, \varepsilon_3)$ are constants. Below we omit the time factor. We seek for solutions of Maxwell's equations in the whole space.

The electromagnetic field $\mathbf{E}, \mathbf{H}$ satisfies Maxwell's Eqs. (1.1), the condition of continuity of tangential field components at the points $x = 0$, $x = h$ on the media interfaces, and the radiation condition at infinity: the electromagnetic field decays as $|x| \to \infty$ in the domains $x < 0$ and $x > h$.

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