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The Hicks Property for a Variational Problem on a Graph

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Abstract—In this paper we use the Hicks property for a variational problem on a graph. For an elastic system defined on a graph we state a well-posed problem which implies the definition and the study of the influence function.

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We consider a positively invertible problem similar to the Leont'ev model

$$u = Au + f \ \ (u, f \in R^n).$$

According to the Hicks property, which is rather popular in Mathematical Economics [1, 2], this problem reveals a strict monotone dependence on the input parameters.

For an indecomposable matrix A (≥ 0) the increase of f only in one component leads not only to the increase of a solution (which follows from the positivity of $(I - A)^{-1}$), but also to its maximal growth in the same component. The extension of this property on more general classes of problems implies the analysis of the influence of a local (δ-shaped) disturbance of the input data on the change of the initial state of the object at the same point. In a more general situation the connection between the disturbed parameter $f(x)$ and the investigated state $u(x)$ can have the following explicit form:

$$Lu = f \tag{1}$$

(this usually takes place in Mathematical Physics). Then the described question implies the study of the equation

$$Lu = \delta(x - \xi)$$

and the analysis of its "fundamental solution" $H(x, \xi)$ on the diagonal. This question is more interesting in the case, when the explicit dependence in the form (1) is not initially stated, for example, in the absence of a standard functional space, where one can describe Eq. (1) and its Green function in classical terms.

The notion of the influence function can help to do without the explicit representation of dependence (1); we understand the influence function as the reaction $K(x, \xi)$ of the initial object on the unit disturbance at the point $x = \xi$. Then under the external load with the density $f(s)ds = dF$ the object deformation caused by this local force takes the form $dh(x) = K(x, s)dF(s)$. The total deformation is defined by the integral over the parameterizing domain Γ

$$h(x) = \int_\Gamma K(x, s)dF(s). \tag{2}$$

The purely intuitive motivation stated above does not allow us to study $K(x, \xi)$ in detail.

Below we correctly state the problem on the influence function for an elastic system defined on a graph and substantiate the Hicks property for the maximum of $K(x, \xi)$ with respect to x.

1. Let Γ be a connected spatial grid (a geometric graph) in R^n.

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