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COXETER TRANSFORMATION AND THE ACYCLICITY CONDITION

V.A. Kolmykov

Introduction. The Coxeter transformation is the superposition of pseudoreflections in a collection of hyperplanes. The data for the system of pseudoreflections form a matrix, for which there corresponds the (associated) Coxeter transformation. One can assign to the matrix of data a geometric object, the carrier consisting of “points” and “marked relations” (the latter symbolize nonzero elements of the matrix); Coxeter graphs and Dynkin diagrams ([1], Chap. V; [2]; [3], Chap. I) are special cases of this construction.

In [4], we associated to the matrix of data a quasiquiver, which allowed us to strengthen a statement from [1] (p. 175), where the carrier is a graph-forest. Can one further strengthen this result by means of weakening the condition that “the carrier is a quasiquiver”? In this paper, we show (Theorem 2) that this is impossible. What is more, the class of quasiquivers is “canonical”: quasiquivers are characterized by means of Coxeter transformations.

1. The Coxeter transformation and quivers. Let A be a complex $n \times n$ matrix. If $U = U(i, j)$ is a condition imposed on i and j, then by $A_{(U)}$ will be denoted the matrix obtained from A in the following way: all the elements whose indices satisfy the condition U remain the same and the other are replaced by zero. For example, if k is a fixed number, then $A_{(j=k)}$ is obtained from A by replacing all elements by zero except for the elements situating in the k-th column. Let $R_k(A) = I - A_{(j=k)}$. Denote

$$C(A) = R_1(A)R_2(A)\dots R_n(A).$$

The Coxeter transformation related to a matrix A is the transformation of the space of rows $x \mapsto xC(A)$.

An oriented graph is a pair (I, E), where I is a finite set (of vertices), $E \subseteq I^2$. In this paper, by the carrier O_A of a matrix A we mean the oriented graph defined as follows. Consider an arbitrary n-element set I and an arbitrary bijection $\mu : \{1, \dots, n\} \to I$, and define E to be the set of pairs $(\mu(i), \mu(j))$ such that $a_{ij} \neq 0$.

An element $(u, v) \in E$ is called an arrow with origin u and end v. A contour of length $k \in \mathbf{Z}/k\mathbf{Z}$ in an oriented graph is a sequence $\{e_1, \dots, e_k\}$ of different arrows such that the end of e_i is the origin of e_{i+1} and no vertex is the end of two arrows of this sequence. A quiver is an oriented graph without contours. We define a *quasiquiver* to be an oriented graph which has no contours of length ≥ 3.

The conditions concerning the occurrence of contours, the boundedness of their lengths and so on are called the acyclicity conditions.[1] At the present time, the progress achived in the study of Coxeter transformations depends substantially upon some acyclicity conditions imposed on the objects of study.

[1]In the case of a nonoriented graph, the notion of a contour is replaced by that of cycle.