Black holes, wormholes and instantons with NUT

GRR-15 KAZAN

D.V. Galtsov
Moscow State University

Based on common work with G. Clement:

1. Wormholes with NUT
2. Quantized rotating Taub-bolt instantons [in preparation]

Black hole with NUT

$$ds^2 = -f\left(dt + 2n(\cos\theta + c)\,d\varphi\right)^2 + \frac{dr^2}{f} +$$

$$(r^2+n^2)(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

c – arbitrary constant

$$f = \frac{r^2 - 2mr - n^2}{r^2+n^2}$$

$$f(r_\pm) = 0 \Rightarrow r_\pm = M \pm \sqrt{M^2+n^2}$$

Newman, Unti, Tamburino (1963) $r > r_+$

Black hole "outer" region

Taub (1951) - Cosmological model

Misner (1963) — Taub solution as $r_- < r < r_+$ of NUT

Ernst description:

$$\mathcal{E} = f + i\chi;$$ n - "magnetic" mass

$dt + \omega_i dx^i \to d\omega = \star d\chi$ in 3d

Asymptotically

$$f = 1 - \frac{2m}{r} + O\left(\frac{1}{r^2}\right)$$

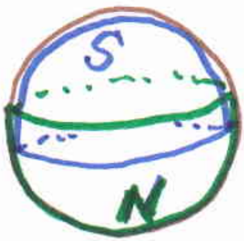
$$\chi = \frac{2n}{r} + O\left(\frac{1}{r}\right)$$

MAGNETIC MONOPOLE analogy:

$$A_\mu dx^\mu = -P(\cos\theta + c)\, d\varphi \quad \&$$

Tetrad $A_{\hat{\varphi}} = -\dfrac{P(\cos\theta + c)}{\sin\theta}$ Singular at $\theta = 0, \pi$

Dirac string



$A^S: c = -1$

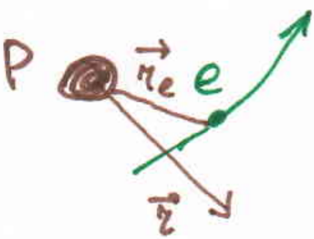
$A^N: c = 1$

$A^S = A^N + d\chi, \quad \chi = 2P\varphi$

$$\psi_e \to e^{ie\chi}\psi_e \Rightarrow 2eP = n \in \mathbb{N}$$

Dirac quantization

Field contribution to angular momentum:



$$\vec{J} = \vec{L} + \vec{S}\,; \quad \vec{L} = \vec{r}_e \times \vec{p}_e$$

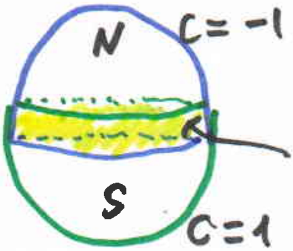
$$\vec{S} = \vec{r}_e \times \int \frac{\vec{E}_e \times \vec{B}_P}{4\pi}\, d^3x = eP\,\hat{r}_e$$

NB: NUT metric in KK interpretaiton gives Monopole KK vector &

(Gross-Perry, Sorkin)

MISNER-DIRAC String

$$-f\left(dt - 2n[\cos\theta + C]\,d\varphi\right)^2$$



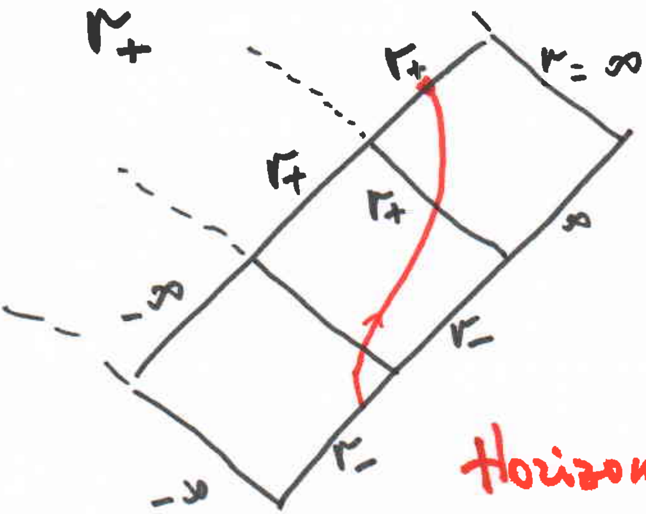
$$t^N = t^S + 4n\varphi$$

Misner: φ periodic with 2π

t periodic with $8\pi n$

eliminate string, converting (it, θ, φ) manifold to S^3

But: Continuation throug the horizon



Geodesic terminates when it passes through the horizon the second time

Horizons cause geodesic incompleteness (Misner, Taub)

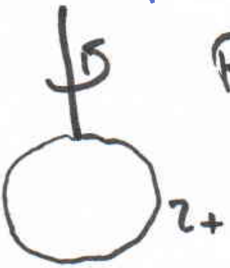
Kagramanova, Kunz .... '10

Bonnor interpretation

$$-f(dt - 2n[\cos\theta + 1]d\varphi)^2$$

regular at South →

then physical singularity at $\theta = 0$

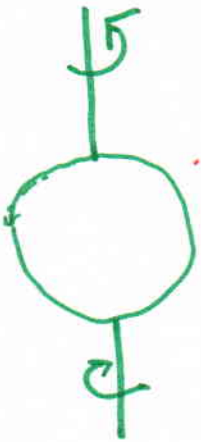


Rotating rod

But: Angular momentum of TAUB-NUT metric is zero

(Manko & Ruiz 2005)

$\Rightarrow$ two rods $(dt - 2n\cos\theta\, d\varphi)^2$



Counterrotating

Rods should have negative mass!

NUT-s in Astrophysics

Whether magnetic mass may exist in nature? (gravitational monopole problem)

Lynden-Bell et al ... + ...

suggest to make search by microlensing

⇒ extra shear due to specific form of geodesics

to lie on cones

[magnetic mass instead of negative mass rods]

Geodesics in Scw-NUT

Killing vectors: $K^{(t)}$, $\vec{K} = (K^{(x)}, K^{(y)}, K^{(z)})$

$$K^{(\pm)} = K^{(x)} \pm i K^{(y)}$$

$$K^{(t)} = \partial_t \,; \quad K^{(z)} = \partial_\varphi$$

$$K^{(\pm)} = e^{\pm i\varphi}\left(\pm i \partial_\theta - \mathrm{ctg}\,\theta\, \partial_\varphi - \frac{2n}{\sin\theta}\partial_t\right)$$

$$\vec{K} \in so(3)$$

Motion integrals: $x^\mu = (t, r, \theta, \varphi)$

$$x^\mu = x^\mu(\tau) \,; \quad \dot{x}_\mu \dot{x}^\mu = \epsilon \left(= \begin{matrix}1\\0\end{matrix}\right)$$

$$E = \dot{x}_\mu K^{(t)\mu} = f(\dot{t} - 2n\cos\theta)$$

$$\vec{J} = \dot{x}_\mu \vec{K}^\mu = \vec{L} + \vec{S}$$

$$\vec{L} = (r^2+n^2)(\dot{\theta}\vec{e}_\varphi - \sin\theta\,\dot{\varphi}\,\vec{e}_\theta) \,; \quad \vec{L}\cdot\vec{e}_r = 0$$

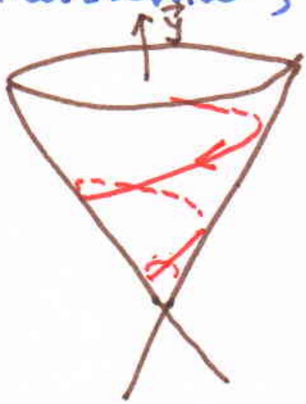
$$\vec{S} = 2nE\,\vec{e}_r$$

$$\vec{L}\cdot\vec{S} = 0 \,; \quad J^2 = L^2 + S^2$$

$\vec{J}\cdot\vec{e}_r = 2nE$, or choosing the frame

$$\vec{J} = (0, 0, J) \,, \quad \cos\theta = \frac{2nE}{J} = \text{const}$$

Thus, motion is not flat as in Scwharzschild, but conical



let $x^{\mu\prime} = (r^2+n^2)\dot{x}^\mu$

$\Downarrow$

$$\varphi' = \frac{J - 2n\cos\theta}{\sin^2\theta}$$

Radial equation

$$\dot{r}^2 + \underbrace{f(r)\left(\frac{\ell^2}{r^2+n^2} + \epsilon\right)}_{U(\vec{r})} = E$$

$$\ell^2 = \vec{L}^2 = \text{const}\,, \quad \text{since } \vec{S}^2 = \text{const}$$

$$f = \frac{(r-r_+)(r-r_-)}{r^2+n^2}$$

$$U = \frac{P_4(r)}{(r^2+n^2)^2}\,; \quad U(r_+) = 0$$

$$U(\infty) = \epsilon$$

- Scattering
- Bound states
- plunging into the hole

Entropy of black holes with NUT

Hawking-Hunter, Mann ...

Computing via tree approximation of quantum gravity

$$Z = \mathrm{Tr}\, e^{-\beta E} = \int \mathcal{D}(g)\, e^{-S[g]} \simeq e^{-S_{cl}}$$

$$\ln Z = S - \beta E \qquad \beta = \frac{1}{T_H}$$

$$S = S_{Beck} + \frac{A_{MS}}{4 \ell_{pl}^2} - \beta H_{MS}$$

MS – Misner string

A – renormalized area

H – Hamiltonian

S_{cl} – classical action of euclidean solution – instanton

Generalization for AdS-NUT $\Rightarrow$

Thermodynamics in finite volume

$\Rightarrow W = E + PV$

Taub-NUT black holes with matter

Einstein-Maxwell: Brill (1963)

EM-dilaton-axion: DG & Kechkin (1994)

Reissner-Nordström-NUT:

$$ds^2 = -f(dt - 2n\cos\theta)^2 + \frac{dr^2}{f} + (r^2+n^2)(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

$$f = \frac{r^2 - 2mr + q^2 - n^2}{q^2+n^2}$$

$$r_\pm = +m \pm \sqrt{m^2+n^2-q^2}$$

Horizon exists if $q^2 < m^2+n^2$.

- Geodesics similar
- Continuation through the horizon – similar problem
- What about $q^2 > m^2 + n^2$ – super-critical
  – No horizon ⇒ less problems

Traversable wormhole without exotic matter (G. Clement, DG)

Horizonless solution of Einstein-Maxwell with NUT:

Source: 1) electric charge (possibly magnetic charge

2) magnetic mass (NUT) (possible ordinary mass)

with $Q^2 + P^2 > m^2 + n^2$

the same metric, with

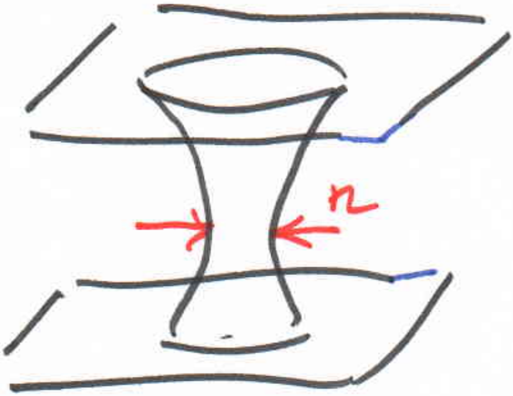
$$f = \frac{(r-m)^2 + b^2}{r^2 + n^2}$$

positive definite

$$b^2 = Q^2 + P^2 - m^2 - n^2$$

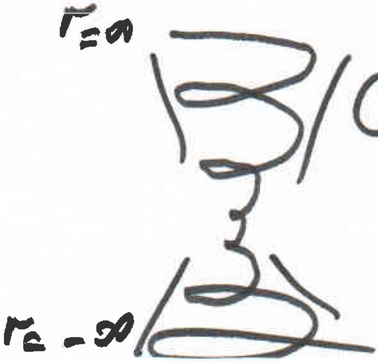
Geodesics in the wormhole

$r \in (-\infty, \infty)$, $r = 0$ - regular



$$ds^2 = -\frac{(r-m)^2+b^2}{r^2+n^2}\left(dt - 2n\cos\theta\, d\varphi\right)^2 + \frac{r^2+n^2}{(r-m)^2+b^2}dr^2 + (r^2+n^2)(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

In the frame $\vec{J} = (0, 0, J)$



Cone: $\cos\theta = \frac{2nE}{J}$

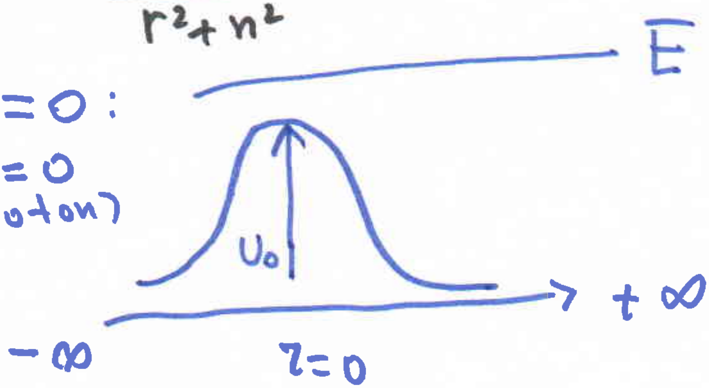
Spiralling trajectories

[no purely radial motion]

Radial potential

$$U = \frac{(r-m)^2+b^2}{r^2+n^2}$$

$m=0$:
$\epsilon = 0$
(photon)



$$U_0 = \frac{b^2\ell^2}{n^4}$$

$$E > \frac{b^2\ell^2}{n^4} \quad \text{traversing}$$

Similar to Ellis wormhole

Instantons

Classification – Hawking, Gibbons et al

ALF: 1+3 ALF ALE

∂_t – killing

$$\|\partial_t\| = 0 \Rightarrow \begin{cases} \text{on } M_d & d=0 \\ \text{or} & d=2 \end{cases}$$

$d=0$ – NUT-s

$d=2$ – bolt-s

Bolt corresponds to euclidean black hole horizon

ALE – genuin 4d $\Rightarrow$ Eguchi Hanson

Multinistantons ...

< 80

In 80-ies – rotating instantons

> 98 – AdS instanton

> 2009 – Using to 5d [B holes & Brings]

Taub NUT and Taub-Bolt

Hawking 1977   Page (1978)

$\psi \in [0, 4\pi]$ (S^3) or $\frac{4\pi}{s}$ lens spaces

$$ds^2 = 4n^2 f(d\psi + \cos\theta\, d\varphi)^2 + \frac{dr^2}{f}$$
$$+ (r^2 - n^2)(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

$$f = \frac{r^2 - 2mr + n^2}{r^2 - n^2} = \frac{(r - r_+)(r - r_-)}{(r - N)(r + N)}$$

$r = n$ point! $(n \to in)$ $2n\psi \to it$

Curvature invariants (right, left)

$$C = -2\frac{(m+n)}{(r+n)^3} \;;\quad \bar{C} = -2\frac{(m-n)}{(r-n)^3}$$

$f = 0$ — fixed point set for ∂_t

$\Downarrow r_0$ $d = 0$ $d = 2$

$r_0 = n$ anti self dual

self dual metric   for $m = n$ (similarly $m = -n$)

then $\bar{C} = 0$, while C - nonsingular

if $s = 1$

$\Rightarrow$ Taub-Nut instantons

Taub-bolt: non self dual

$m > n$ $\cancel{}$, $f(r_+) = 0$

r_+ surface of finite area

Since $r_+ = m + \sqrt{m^2 - n^2} > m$
and $m > 0$
$\Rightarrow r_+ > n$
no curvature singularity

But:
$f^{-1} dr^2 + 4n^2 f d\psi^2$
can have conical singularity
Avoided if $\left[\psi \in [0, \frac{4\pi}{5}]\right]$

$$\frac{4\pi}{5} \, n \frac{(r_+ - r_-)}{r^2 - n^2} = 2\pi \qquad [T_H = 8\pi n]$$

$$\Rightarrow \quad m = \frac{5}{4} n$$

$$f = \frac{r^2 - \frac{5}{2} r n + n^2}{r^2 - n^2}$$

Page Bolt $f(r_+) = 0$

$$r_+ = \left(\frac{5}{4} + \sqrt{\frac{25}{16} - 1}\right) n = 2n$$

Kerr - Taub - Bolt

$$ds^2 = \frac{\Delta}{\Sigma}\left(dt + 2n\cos\theta - a\sin^2\theta + \underline{\underline{C}}\right)^2$$

$$+ \Sigma\left(\frac{dr^2}{\Delta} + d\theta^2\right) + \frac{\sin^2\theta}{\Sigma}\left(P_r d\varphi + a\, dt\right)^2$$

$$P_r = r^2 - n^2 - a^2 + Ca$$

$$\Sigma = r^2 - (n + a\cos\theta)^2$$

$$\Delta = (r^2 - 2Mr + n^2 - a^2)$$

C - arbitrary for $a = 0$
[defining position of Misner string]

But for $a \neq 0$ C is not arbitrary and "quantized":

$$C = 2kn \quad , \quad n\text{ - odd integer}$$

Previously known rotating instantons do not fall into this family

① Rotating Taub-NUT-s

- become Bolts with $m=n$

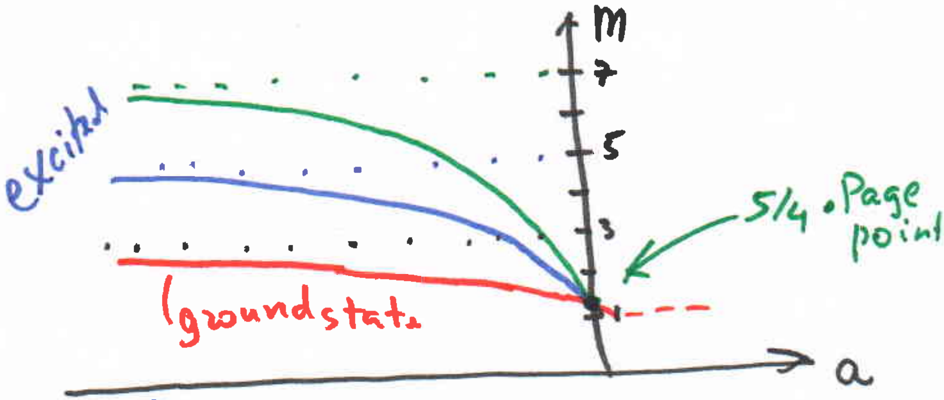
$$r_+ = n + |a| \; ; \quad a<0$$

- exist excitations such that

$m_k(a=0) = n$, but

$m_k \to (k+2)n$ at $a \to -\infty$

② Rotating Page bolts



$$\tilde{m}_k(a) = \frac{5}{4} n$$

$$\tilde{m}_k(-\infty) = (k-2)n$$

Conclusions

1) Third version of Lorentzian ALF without horizons

2) Traversable wormholes between 2 ALF metrics exist without exotic matter

[price: Misner string ⇒ interpretation?]

3) Horizonless Taub-Nut avoid Misner-Taub problem of non-Hausdorf

4) Rotating instantons with NUT exist in 2 versions
- rotating Taub-Nuts [Bolt1]
- rotating Page Bolts

5) Forming asymptotically equidistant spectrum of mass