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PROJECTING PROCEDURES OF NONLOCAL IMPROVEMENT OF LINEARLY CONTROLLABLE PROCESSES

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1. Introduction

For computation of optimal control problems the iteration gradient type techniques of needle variation are usually used [1]–[3]. The improvement of the objective functional at each iteration of these methods is attained only locally, i. e., in a sufficiently small neighborhood of weak or needle variation of control. In some classes of problems, we can omit the local search of the improving control. This is essential for improving the computational efficiency. In [4] and [5], proposed are the procedures for nonlocal improvement of controls in control systems which are linear and quadratic with respect to the state, respectively. The charge for the nonlocal improvement in linear as respects the state systems is the necessity to solve the Cauchy problems for the system of differential equations with the discontinuous right-hand side. For the quadratic as respects the state systems, the charge is the boundary-value problem for the system of differential equations with the discontinuous as respects the state right-hand side. This boundary-value problem is much simpler relative to the smoothness characteristics than that of the maximum principle. For the case of linear as respects the state optimal control problem, the boundary-value problem of control improvement is reduced to solving the Cauchy problems and the corresponding improvement procedures [5] become equivalent to those described in [4] (pp. 14–15).

In this paper, we propose new procedures for nonlocal improvement of control for the linear as respects the state optimal control problem

$$\Phi(u) = \varphi(x(t_1)) + \int_T \{F_0(x(t),t) + \langle F_1(x(t),t), u(t)\rangle\}dt \to \min_{u\in V}, \tag{1}$$

$$\dot{x}(t) = b(x(t),t) + A(x(t),t)u(t), \quad x(t_0) = x^0,\ u(t) \in U, \quad t \in T = [t_0,t_1]. \tag{2}$$

Here $x(t) = (x_1(t),\dots,x_n(t))$ is the state vector, $u(t) = (u_1(t),\dots,u_m(t))$ is the control vector. The matrix function $A(x,t)$, the vector functions $b(x,t)$, $F_1(x,t)$ and the functions $\varphi(x)$, $F_0(x,t)$ are quadratic as respects x with the coefficients which depend on t continuously on the set $R^n \times T$. As the set of admissible controls $u(t)$, $t \in T$, we consider the class V of piecewise continuous functions which take on values from the convex closed set $U \subset R^m$. We understand the equality of admissible controls accurate to set of the null measure on the interval T. The initial state x^0 and the control period T are given. We use the following notation: $\langle x,y\rangle$ is the scalar product of the vectors x and y, $\|x\|$ is the norm of the vector x in the Euclidian space, A^T is the transposed matrix A, $\Delta_z q(x,u,\dots) = q(z,u,\dots) - q(x,u,\dots)$, $\Delta_v q(x,u,\dots) = q(x,v,\dots) - q(x,u,\dots)$, and $\Delta_{z,v} q(x,u,\dots) = q(z,v,\dots) - q(x,u,\dots)$ are partial increments of the function q with a finite number of arguments, q_x, q_u, q_{xx}, q_{uu}, and q_{xu} are partial derivatives of the function q of the first and the second order as respects the arguments x and u, respectively.