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NORMALIZATIONS AND AFFINE CONNECTIONS ON DISTRIBUTIONS OF CONFORMAL SPACE

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In this article we intrinsically construct complete invariant normalizations of an m-dimensional distribution $\mathfrak{M}$ in an n-dimensional proper conformal space C_n by elements of the second-order differential neighborhood, and study the properties of affine connections induced on $\mathfrak{M}$. We will use the following index ranges:

$$\lambda,\mu,\rho=\overline{0,n+1};\quad \overline{I},\overline{K},\overline{L}=\overline{0,n};\quad I,K,L=\overline{1,n};\quad i,j,k,l,s,t=\overline{1,m};$$
$$\overline{i},\overline{j},\overline{k}=\overline{0,m};\quad \alpha,\beta,\gamma=\overline{m+1,n}.$$

1. Let us consider a proper conformal space C_n referred to a moving semi-isotropic frame $R=\{A_\lambda\}$ (see [1]) consisting of two points A_0, A_{n+1} and n hyperspheres A_K of real nonzero radius, passing through these points. Let us denote by $g_{\lambda\mu}$ the scalar product $(A_\lambda A_\mu)$ of frame elements, then we have (see [2])

$$\|g_{\lambda\mu}\|=\begin{Vmatrix}0&0&1\\0&g_{IK}&0\\1&0&0\end{Vmatrix},\quad g_{\lambda\mu}=g_{\mu\lambda}, \tag{1}$$

and since C_n is a proper conformal space, the matrix $\|g_{IK}\|$ is nondegenerate and positive definite.

It is known (see [1], [2]) that to each point in C_n a point of an oval real hyperquadric Q_n^2 in the projective space P_{n+1} corresponds, which is called the Darboux hyperquadric. Q_n^2 is given by the equation

$$g_{IK}x^Ix^K+2x^0x^{n+1}=0,$$

where x^λ are coordinates of points $P\in\mathrm{C}_n$ with respect to the frame R.

An infinitesimal transformation lying in the conformal group $\mathfrak{L}$ (i.e., in the stationary group of the absolute $Q_n^2\subset\mathrm{P}_{n+1}$) displace elements of the conformal frame R. The principal part of this displacement is given by the differentials dA_λ, which are hyperspheres. These differentials are expressed as linear combinations of the elements of R,

$$dA_\lambda=\omega_\lambda^\mu A_\mu, \tag{2}$$

where the Pfaff differential forms ω_λ^μ depend on the parameters of the group $\mathfrak{L}$. The number of linearly independent forms among the forms ω_λ^μ equals $(n+1)(n+2)/2$, the number of independent parameters of this group. The following structure equations give the integrability conditions for system (2):

$$D\omega_\lambda^\mu=\omega_\lambda^\rho\wedge\omega_\rho^\mu.$$

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