ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2010, Vol. 54, No. 2, pp. 57–65. © Allerton Press, Inc., 2010.
Original Russian Text © S.Ya. Serovaiskii, 2010, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2010, No. 2, pp. 66–76.

Differentiation of Operators and Optimality Conditions in Category Interpretation

S. Ya. Serovaiskii[1*]

[1] *Kazakh National University, ul. Masanchi 39/47, Almaty, 050012 Republic of Kazakhstan*
Received October 10, 2007; in final form, October 27, 2008

Abstract—The general extremum theory essentially uses properties of operator derivatives. As an example we consider a system described by a nonlinear elliptic equation. In this system with large values of the nonlinearity parameter and the domain dimension the control-state mapping is not Gâteaux differentiable. For this reason one cannot immediately differentiate the optimality criterion and establish the necessary optimality conditions by classical methods. However the mentioned mapping is extendedly differentiable. This allows one to obtain optimality conditions imposing no constraints on system parameters. Concluding the paper, we interpret the optimality conditions with classical and extended derivatives within the theory of categories.

DOI: 10.3103/S1066369X10020076

Key words and phrases: *optimality conditions, operator derivatives, nonlinear elliptic equation, categories.*

1. THE CLASSICAL DIFFERENTIATION

In the general theory of extremum and general schemes for obtaining the necessary optimality conditions one essentially uses properties of operator derivatives (for example, [1–5]). Assume, in particular, that the state equation of a control system is given

$$Ay = v, \tag{1}$$

where $A : Y \to V$ is a continuously differentiable state operator, Y and V are Banach spaces, v is a control, and y is a state function of the system. We also assume that for any $v \in V$ Eq. (1) has a unique solution $y = y(v)$ in the space Y. The optimality criterion $I : V \to \mathbb{R}$ is defined in accordance with the equality $I(v) = J(v) + K[y(v)]$, where $J : V \to \mathbb{R}$ and $K : Y \to \mathbb{R}$ are continuously differentiable functionals. We state the following optimal control problem:

Problem P. *Find a point $v \in V$ that minimizes the functional I in the space V.*

It is not difficult to obtain the following assertion.

Theorem 1. *If the derivative of the operator A at the point $y = y(v)$ is invertible, where v is a solution to problem P, then the following equality is valid*:

$$J'(v) + p = 0, \tag{2}$$

p is a solution to the equation

$$[A'(y)]^* p = K'(y). \tag{3}$$

[*] E-mail: serovajskys@mail.ru.