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The Weighted L^1-Integrability of Functions and the Parseval Equality with Respect to Multiplicative Systems

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Abstract—In this paper we prove necessary and sufficient conditions for the weighted L^1-integrability of functions defined on $[0,1)$ in terms of Fourier coefficients with respect to a multiplicative system of bounded type. These results are counterparts of trigonometric ones obtained by M. and S. Izumi and M. M. Robertson.

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INTRODUCTION

Let $\{p_n\}_{n=1}^{\infty}$ be a sequence of natural numbers such that $2 \leqslant p_n \leqslant N$, $n \in \mathbb{N}$. Define $m_0 = 1$ and $m_n = p_1 \dots p_n$ for $n \in \mathbb{N}$. Then each $x \in [0,1)$ can be decomposed as

$$x = \sum_{n=1}^{\infty} x_n/m_n, \quad x_n \in \mathbb{Z}(p_n) := \mathbb{Z} \cap [0, p_n). \tag{1}$$

Decomposition (1) is unique, if for $x = k/m_j$, $k, j \in \mathbb{N}$, $0 < k < m_j$, we use a decomposition with a finite number of nonzero x_n. If $x, y \in [0,1)$ take form (1), then by definition we have $x \ominus y = z = \sum\limits_{n=1}^{\infty} z_n/m_n$, where $z_n = x_n - y_n \pmod{p_n}$, $z_n \in \mathbb{Z}(p_n)$. We similarly define $x \oplus y$.

Each $k \in \mathbb{Z}_+$ is uniquely representable as

$$k = \sum_{j=1}^{\infty} k_j m_{j-1}, \quad k_j \in \mathbb{Z}(p_j). \tag{2}$$

For $x \in [0,1)$ and $k \in \mathbb{Z}_+$ written as (1) and (2), respectively, we define $\chi_k(x) = \exp\Bigl(2\pi i \sum\limits_{j=1}^{\infty} x_j k_j / p_j\Bigr)$. It is known that the system $\{\chi_k(x)\}_{k=0}^{\infty}$ is orthonormal on $[0,1)$ and complete in $L[0,1)$, while $\chi_n(x \oplus y) = \chi_n(x)\chi_n(y)$ and $\chi_n(x \ominus y) = \chi_n(x)\overline{\chi_n(y)}$ for almost all $y \in [0,1)$ with fixed $x \in [0,1)$ and $n \in \mathbb{Z}_+$. If $k, l \in \mathbb{Z}_+$ are written in the form (2), then $k \oplus l := r = \sum\limits_{i=1}^{\infty} r_i m_{i-1}$, where $r_i = k_i + l_i \pmod{p_i}$, $k_i \in \mathbb{Z}(p_i)$. We similarly define $k \ominus l$.

For any $[0,1)$ we have the equalities

$$\chi_k(x)\chi_l(x) = \chi_{k \oplus l}(x), \quad \chi_k(x)\overline{\chi_l(x)} = \chi_{k \ominus l}(x).$$

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