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# A PROBLEM IN THE THEORY OF GRADUATED FORMATIONS

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All groups considered in this article are assumed to possess finite composition series. Recall that a class $\mathfrak{F}$ is called a formation (see [1]) if $\mathfrak{F}$ is a homomorph (i. e., every factor group of any group from $\mathfrak{F}$ belongs to $\mathfrak{F}$) and from $G/N \in \mathfrak{F}$ and $G/R \in \mathfrak{F}$ it follows that $G/N \cap R \in \mathfrak{F}$. In applications of the theory of formations the so-called graduated formations are of a special importance; they were first introduced by L.A. Shemetkov in [2] (see also [3], Chap. 1).

Let $\mathfrak{G}$ stand for a class of all groups. A mapping $f : \mathfrak{G} \mapsto \{\text{classes of groups}\}$ such that $f(1) = \mathfrak{G}$ and $f(G_1) = f(G_2)$ for $G_1 \cong G_2$ is called a screen (see [2]) if $f(G)$ is a formation and we have $f(G) \subseteq f(N) \cap f(G/N)$ for any group $G$ and any its normal subgroup $N$.

For a screen $f$ we denote by $\langle f\rangle$ the class of all groups $G$ such that $G/C_G(H/K) \in f(H/K)$ for every principal factor $H/K$. A formation $\mathfrak{F}$ is called a graduated formation (see [2]) if the equality $\mathfrak{F} = \langle f\rangle$ takes place for a certain screen $f$.

A screen $f$ is said to be a composition screen (see [2]) if, for any group $G \neq 1$, we have $f(G) = \bigcap f(H/K)$, where $H/K$ runs over all composition factors of $G$. A formation $\mathfrak{F}$ is called a composition formation if $\mathfrak{F} = \langle f\rangle$ for a certain composition screen $f$.

The composition formations are of particular importance in applications (see [3], [4]), and the question: "What are the conditions under which a given formation is a composition formation?" also is of interest. In accordance with the famous Baer theorem (see [5], p. 373), a nonempty formation of finite groups $\mathfrak{F}$ is a composition formation if and only if a finite group $G$ belongs to $\mathfrak{F}$ when $G/\Phi(R(G)) \in \mathfrak{F}$ (here $R(G)$ is the radical of $G$, i. e., the product of all its normal solvable subgroups).

In 1995, at the Gomel Algebraic Seminar, O.V. Mel'nikov stated a conjecture that every graduated formation of finite groups is a composition formation. This conjecture is partially confirmed by many known examples of graduated formations and by the results in [6], concerning Mel'nikov's problem.

In this article an example is constructed which demonstrates that, in general case, the answer to O.V. Melnikov's conjecture is negative. However, first of all we need to analyze and simplify the proper definition of the graduated formation.

Let a function $f$ assign to each elementary group $G$ a formation $f(G)$. We will say that the function $f$ is an $e$-function if $f(H) = f(T)$ for any two elementary groups $H$ and $T$ such that $H \cong T$. Let $EF(f)$ be a class of all groups $G$ such that $G/C_G(H/K) \in f(H/K)$ for each principal factor $H/K$ of $G$. By the definition, all the trivial groups belong to the class $EF(f)$.

**Proposition 1.** *For any e-function $f$ the class $EF(f)$ is a formation.*

**Proof.** Let $G \in EF(f)$ and assume that $N$ is an arbitrary normal subgroup of $G$. Let $(H/N)/(K/N)$ be an arbitrary principal factor of $G/N$. Then $H/K$ is a principal factor of $G$. Hence, by the assumption on $G$, we have $G/C_G(H/K) \in f(H/K)$. By lemma 2.8 in [4], we have $C_{G/N}((H/N)/(K/N)) = C_G(H/K)/N$. Therefore $(G/N)/C_{G/N}((H/N)/(K/N)) =$