Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 44, No. 9, pp. 72–76, 2000 *UDC 517.544*

REDUCTION OF THE SOLUTION OF INTERIOR INVERSE BOUNDARY VALUE PROBLEM TO INTEGRAL EQUATION IN CASE OF ANGULAR POINTS ON DESIRED AND KNOWN CONTOURS

Ye.A. Shirokova

In [1] the solution of the interior inverse boundary value problem for the parameter s was reduced to solving integral Fredholm equation for the cases of the classical statement of problem (see [2], [3]) where the known boundary values $w(s)$ are a function with Hölder derivative, and also for a generalized statement of the same problem in the case where both $w'(s)$ and its inverse value are Lebesgue-integrable. In the classical statement it was provided that $w'(s)$ differs from zero; this resulted in that the desired contour was smooth. In the generalized statement the mentioned restrictions were partially discharged; however, the resulting solution was belonging to a wider class of functions and thus was not revealing the behavior of the solution in a neighborhood of points, where the violation of classical conditions take place. In item 1 of this article we admit that in some points the function $w'(s)$ turns into either zero, or ∞ and give a solution of the problem via reduction to an integral equation. In item 2 we obtain weaker conditions on the initial data for the generalized statement. In addition, in contrast to [1], we admit an angular point on the known contour. We give restrictions with which the problem can be reduced to solving an integral equation.

1. Let the conditions imposed on $w(s)$ in [1] in the case of the classical statement of the problem be violated: In spite of the smoothness of the known contour Γ_w, the desired contour Γ_z at the point corresponding to the parameter s_0, forms an angle which equals $\frac{\pi}{\gamma}$. In accordance with [3], [4], the latter means that

$$w(s) = w(s_0) + \operatorname{sgn}(s - s_0)|s - s_0|^\gamma [u_1(s) + iv_1(s)], \quad \gamma \geq 1/2, \quad s \in [0, l], \tag{1}$$

where $u_1^2(s_0) + v_1^2(s_0) \neq 0$, $u_1'(s), v_1'(s) \in C_\alpha[0, l]$, $\alpha \in (0, 1]$. We shall suppose that the contour Γ_w possesses the unique point with this property. As concerns the remaining restrictions upon $w(s)$, they are same as in [1]: The contour Γ_w is simple and closed, $w'(s) \neq 0$, $s \in [0, l] \setminus \{s_0\}$. We shall write for $\phi \in C_\alpha[0, l]$:

$$\|\phi\|_{H_\alpha} = \sup_{t_1, t_2 \in [0, l]} |\phi(t_1) - \phi(t_2)|\, |t_1 - t_2|^{-\alpha}, \quad \|\phi\|_C = \max_{t \in [0, l]} |\phi(t)|.$$

Let us introduce a new parameter for the curve $\Gamma_w : \sigma = \operatorname{sgn}(s - s_0)|s - s_0|^\gamma$. Then $s = s_0 + \operatorname{sgn}\sigma|\sigma|^{1/\gamma}$, and after introduction of the new parameter we obtain

$$\widetilde{w}(\sigma) \equiv w(s(\sigma)) = w(s_0) + \sigma[u_1(s_0 + \operatorname{sgn}\sigma|\sigma|^{1/\gamma}) + iv_1(s_0 + \operatorname{sgn}\sigma|\sigma|^{1/\gamma})],$$

$$\frac{d\widetilde{w}}{d\sigma} = u_1(s_0 + \operatorname{sgn}\sigma|\sigma|^{1/\gamma}) + iv_1(s_0 + \operatorname{sgn}\sigma|\sigma|^{1/\gamma}) + $$
$$+|\sigma|^{1/\gamma}[u_1'(s_0 + \operatorname{sgn}\sigma|\sigma|^{1/\gamma}) + iv_1'(s_0 + \operatorname{sgn}\sigma|\sigma|^{1/\gamma})]\tfrac{1}{\gamma}.$$