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The Riemann Boundary-Value Problem on an n-Sheeted Surface Free of Limit Points of Projections of Branch Points Onto $\mathbb{C}$

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Abstract—We obtain solvability conditions and explicit solutions for the Riemann boundary-value problem on an n-sheeted surface in the case when projections of branch points on the complex plane condense only at infinity.

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The Riemann boundary-value problem on a compact Riemann surface is solved by L. I. Chibrikova, R. N. Abdullaev, and E. I. Zverovich ([1, 2]). On a noncompact Riemann surface this problem is solved explicitly in [3] for the case of a finite-sheeted surface such that all its branch points have the maximal order and their projections on the complex plane $\mathbb{C}$ have no limit points in $\mathbb{C}$. In this paper we obtain solvability conditions and explicit solutions for the Riemann boundary-value problem on an n-sheeted surface in the case when branch points can have any orders, and their projections on the complex plane condense only at infinity.

PRELIMINARIES

We assume that R is a Riemann surface of infinite genus, and a holomorphic on this surface function z takes any its value in $\mathbb{C}$ just n times (taking into account the multiplicities). Then the mapping $z : R \to \mathbb{C}$ defines an n-sheeted boundless covering (R, z) of the plane $\mathbb{C}$ with an infinite number of branch points, whose projections have no limit points in $\mathbb{C}$. Let D be a domain in R with a compact complement $R \setminus D$ such that (D, z) is a boundless covering for the domain $G := z(D) \subset \mathbb{C}$. Let G' be the subset of G consisting of those points over which n different points of the covering (D, z) lie, i.e., $G \setminus G'$ consists of projections on G of all branch points of the covering (D, z). Any disk $U \subset G'$ is the bijective image of any of n domains $U_i \subset D$, $i = 1, 2, \dots, n$ under the mapping $z : D \to G$.

Let F be a bounded holomorphic function in D. The function $v(z) := F(q(z))$ (here $q(z)$ is the lifting of the point $z \in \mathbb{C}$ onto the covering (R, z)) is, generally speaking, a multivalued analytic function in G, and its branch points are projections on $\mathbb{C}$ of branch points of the covering (D, z). In any disk $U \subset G'$ the function $v(z)$ admits the separation of n single-valued branches $v_i(z) = F(q(z))$, $q \in U_i$, $i = 1, 2, \dots, n$.

Lemma. *Assume that D is a domain on R with a compact complement such that (D, z) is a boundless covering for the domain $G \subset \mathbb{C}$; let F be a bounded holomorphic function in D. Then in any disk $U \subset G'$ at least two branches of the function $v(z) := F(q(z))$ coincide.*

Proof. We put

$$H(z) = \prod_{1 \le i < k \le n} (v_i(z) - v_k(z))^2, \quad z \in U.$$

This function is analytically continuable along any path in the domain G' and does not change under all possible permutations $v_1, v_2, \dots, v_n$. Consequently, H is a bounded holomorphic function in the domain G' analytically extendable into G.

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