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On Approximation of Dual Spaces and Dual Operators

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Abstract—We study the connection between equations which approximate an initial abstract linear equation and the adjoint one. We prove that the operator which is adjoint to the approximating one approximates the adjoint operator. As examples we consider adjoint linear integral equations and mutually dual linear programming problems.

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The notions of the dual space and the adjoint operator are essentially used in the theory of solving linear integral equations and, more generally, linear operator equations (e.g., [1], Chap. I; [2], Chap. IV). In this paper we study the relationship between equations which approximate a linear operator equation and its dual one. In order to allow more freedom of reasoning, we employ the abstract duality theory ([3], Chap. 8). We use the definitions and terminology of the abstract theory of approximation methods developed in [4].

We prove that the operator dual to an approximating one approximates the dual operator. A similar statement is obtained for mutually dual problems of abstract linear programming. As an example we consider integral Fredholm equations of the 2nd kind.

1. APPROXIMATION AND INTERPOLATION

Let X, Y and $\overline{X}$, $\overline{Y}$ stand for the exact and approximating spaces; let T_X, S_X and T_Y, S_Y be the pairs of approximation and interpolation operators; let $A : X \to Y$ and $\overline{A} : \overline{X} \to \overline{Y}$ denote an exact linear operator and an approximating one (see Figure 1).

$$\begin{array}{ccc} X & \xrightarrow{A} & Y \\ {\scriptstyle T_X}\downarrow\uparrow{\scriptstyle S_X} & & {\scriptstyle T_Y}\downarrow\uparrow{\scriptstyle S_Y} \\ \overline{X} & \xrightarrow{\overline{A}} & \overline{Y} \end{array}$$

Fig. 1.

Here, as in [4], a space $\overline{X}$ approximates a space X, if the approximation and interpolation operators $T_X : X \to \overline{X}$ and $S_X : \overline{X} \to X$ are defined so that $T_X S_X = I$ (I is the identity operator). For a nontrivial approximation, $S_X T_X \neq I$. If spaces $\overline{X}$ and $\overline{Y}$ approximate those X and Y, then we treat any operator $\overline{A} : \overline{X} \to \overline{Y}$ as an operator which approximates the exact one A (we call it an approximation of A). Similarly, given an operator $\overline{A} : \overline{X} \to \overline{Y}$, we treat any operator $\widetilde{A} : X \to Y$ as an operator which

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