Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 41, No. 10, pp.13–23, 1997 *UDC 517.955*

A CAUCHY PROBLEM WITH INITIAL DATA ON DIFFERENT SURFACES FOR A SYSTEM WITH SINGULARITY

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For the case where the initial data for different functions are given on different surfaces, we construct analytical solutions of a quasilinear partial differential equation system when the resulting problem has a concrete singularity of the form u/x or x/u. The necessary and sufficient conditions for existence and uniqueness of the solution of this system in the form of formal power series are given as well as the sufficient condition for the convergence of these series. There is given a generalization of problems under consideration, which must be investigated in constructing the currents of ideal gas, which appear after a shock wave reflected by an axis or by a symmetry center.

For systems of partial differential equations the Cauchy problem (Cp) is well-studied. For such a Cp, in the case where all input data are analytical, the Cauchy-Kovalevskaya theorem (CKt) is valid (see [1], p. 23; [2], p. 50). A generalized Cp (Cp with initial data on different surfaces) was considered [3], [4] and the corresponding generalizations of CKt were proved. For linear integral differential equations with a singularity the Goursat problem was considered and an analog of CKt was proved [5]. If in the posed Cp the determinant of a matrix which stays in front of the vector of the output derivations vanishes, then there arise many various situations. In particular, there takes place a characteristic Cp, for which we have proved the respective analog of CKt (see [6]). In the present article we consider a specific case of Cp with initial data on different surfaces, when the determinant of the respective matrix also vanishes. This case is equivalent to the presence in the problem of a singularity of the form u/x or x/u.

Let us consider the Cauchy problem with initial data on various surfaces for a quasilinear system in the simplest case of two unknown functions depending on two independent variables:

$$A\begin{pmatrix} u_x \\ v_x \end{pmatrix} + b\begin{pmatrix} u_y \\ v_y \end{pmatrix} = \vec{C}. \tag{1}$$

Here A and B are matrices of the dimension 2×2, $\vec{C} \in R^2$, the coefficients of matrices A and B as well as components of the vector $\vec{C}$ are functions of x, y, u, and v. On two different curves $\phi_i(x, y) = 0$, $i = 1, 2$, which intersect at the point (x_0, y_0), we put for the unknown functions the following two initial conditions:

$$\Phi_i(x, y, u, v)\big|_{\phi_i(x,y)=0} = 0, \quad i = 1, 2. \tag{2}$$

Here ϕ_i, Φ_i, $i = 1, 2$, are given functions of their arguments,

$$\phi_i(x_0, y_0) = 0, \quad \Phi_i(x_0, y_0, u_0, v_0) = 0, \quad x_0, y_0, u_0, v_0 \text{— const}.$$

The problem (1), (2) is the Cp with initial data on different surfaces for the quasilinear system. Further for the brevity it is called problem A.

Supported by State Committee of Russia for Higher Education via the Grant Center at Novosibirsk State University.