ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2009, Vol. 53, No. 8, pp. 43–45. © Allerton Press, Inc., 2009.
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Filtered Deformations of the Frank Algebras

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Received April 23, 2007

Abstract—In this paper we prove the rigidity of simple Lie algebras of the Frank series of characteristic 3 with the standard grading of depth 2 with respect to filtered deformations.

DOI: 10.3103/S1066369X09080076

Key words and phrases: *graded Lie algebra, filtered deformation, Spencer cohomology.*

The simple exceptional algebras of the Frank series $T = T(m)$ exist over a field of characteristic 3. Their geometric realization was obtained in papers [1]–[3] (see also [4]). The $\mathbb{Z}$-graded algebra $T(m)$ of depth 2 admits an embedding in the contact algebra $\mathcal{K} = \mathcal{K}(3 : 1, 1, m)$ ([5]). Deformations of an algebra T satisfy the assumptions of the embedding theorem for a contact algebra [6], therefore one can reduce the question about the existence of filtered deformations of the algebra T to the computation of the subgroup $H^1_+(T, \mathcal{K}/T)$ in the first cohomology group. In this paper we prove that $H^1_+(T, \mathcal{K}/T) = 0$ and therefore $T(m)$ has no filtered deformations that are not isomorphic to $T(m)$.

Assume that $\mathcal{O} = \mathcal{O}(1 : m)$ is the divided power algebra in one indeterminate, $W = W(1 : m)$ is the algebra of special derivations of $\mathcal{O}$, $\Omega = \Omega^1$ is the space of differential forms of degree 1, and U is a vector space of dimension 2. Let us fix a skewsymmetric nondegenerate bilinear form $\langle\ ,\ \rangle$ on U and define a linear operator $T_{uu'} \in \mathfrak{sl}(U)$ on U for $u, u' \in U$

$$T_{uu'}(v) = \langle u, v\rangle u' + \langle u', v\rangle u \ \ \forall v \in U.$$

The algebra $T = T(m)$ is $\mathbb{Z}_2$-graded: $T = T_{\bar 0} \oplus T_{\bar 1}$, where $T_{\bar 0} = W \oplus (\mathfrak{sl}(U) \otimes \mathcal{O})$ and $T_{\bar 1} = U \otimes \Omega$. The multiplication in this algebra is defined as follows:

1) $[\ ,\ ] : T_{\bar 0} \times T_{\bar 0} \longrightarrow T_{\bar 0}$:

$$[f_1\partial + G_1 \otimes h_1, f_2\partial + G_2 \otimes h_2] = [f_1\partial, f_2\partial] + G_2 \otimes f_1\partial h_2 - G_1 \otimes f_2\partial h_1 + [G_1, G_2] \otimes h_1h_2,$$

where $f_1, f_2, h_1, h_2 \in \mathcal{O}$, $G_1, G_2 \in \mathfrak{sl}(U)$;

2) $[\ ,\ ] : T_{\bar 0} \times T_{\bar 1} \longrightarrow T_{\bar 1}$:

$$[f\partial + G \otimes h, u \otimes g\mathrm{d}x] = u \otimes \partial(fg)\mathrm{d}x + G(u) \otimes hg\mathrm{d}x,$$

where $f, g, h \in \mathcal{O}$, $u \in U$, $G \in \mathfrak{sl}(U)$;

3) $[\ ,\ ] : T_{\bar 1} \times T_{\bar 1} \longrightarrow T_{\bar 0}$:

$$[u_1 \otimes f\mathrm{d}x, u_2 \otimes h\mathrm{d}x] = \langle u_1, u_2\rangle fh\partial + T_{u_1u_2} \otimes (f\partial h - h\partial f),$$

where $u_1, u_2 \in U$, $f, h \in \mathcal{O}$.

The Frank algebra T has a $\mathbb{Z}$-grading of depth 2: $T = T_{-2} \oplus T_{-1} \oplus T_0 \oplus T_1 \oplus \cdots$, where

$$T_{2i} = \langle x^{(i+1)}\partial, E \otimes x^{(i)},\ F \otimes x^{(i)},\ H \otimes x^{(i)}\rangle,\ \ i \geq -1,\ \ x^{(-1)} = 0,$$
$$T_{2i+1} = \langle e_1 \otimes x^{(i+1)}\mathrm{d}x,\ e_2 \otimes x^{(i+1)}\mathrm{d}x\rangle,\ \ i \geq -1,$$

E, F, H form the standard basis for $\mathfrak{sl}(U)$, and e_1, e_2 is a symplectic basis for the space U.

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