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Abelian Groups With Sufficiently π-Regular Endomorphism Ring

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Abstract—The notion of π-regular endomorphism ring of an abelian group, which generalizes the notion of regular endomorphism ring, was introduced in papers of L. Fuchs and K. Rangaswamy. They described periodic abelian groups with π-regular endomorphism ring and found necessary conditions for an abelian group to have π-regular endomorphism ring. In this paper, we study abelian groups with sufficiently π-regular endomorphism ring, which form a subclass of the class of abelian groups with π-regular endomorphism ring, and find necessary and sufficient conditions for an abelian group to have sufficiently π-regular endomorphism ring.

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This paper is related to Problem 7 from [1]: "To describe reduced mixed abelian groups whose endomorphism rings are regular". Recall that a ring R is called regular (respectively, π-regular) if for each element $x \in R$ there exists an element $y \in R$ such that $xyx = x$ (respectively, such that $x^m y x^m = x^m$ for some $m \in \mathbb{N}$ depending on x). Note that the basic studies of abelian groups with regular (π-regular) endomorphism ring are connected with papers of K. Rangaswami [2], L. Fuchs and K. Rangaswami [3], in which the study of such groups was reduced to the case of reduced groups. A detailed description of these results can also be found in monographs [4] and [5]. A description of torsion-free reduced groups of finite rank with regular endomorphism ring was suggested by S. Glaz and W. C. Wickless in [6]. Necessary and sufficient conditions for a reduced abelian group to have regular endomorphism ring were found in [7]. Abelian groups whose endomorphism rings have regular center were studied in [7] and [8]. In Theorem 1 of this paper we give a description of abelian groups with sufficiently π-regular endomorphism ring.

All groups that appear in the paper are assumed to be abelian. We use the following notation: $\mathbb{Q}$ is the field of rational numbers, the direct sum and the product of groups (rings) are denoted by $\bigoplus$ and $\times$ or $\prod$, respectively, $T(X)$ is the periodic part of a group X, $T_p(X)$ is the p-component of $T(X)$, $E(X)$ is the endomorphism ring of a group X, $\mathrm{Hom}(X, Y)$ is the group of homomorphisms from a group X to a group Y, $X[p^m] = \{a \in X \mid p^m a = 0\}$, $C(R)$ is the center of a ring R, (m, n) denotes the greatest common divisor of integers m and n, P denoted the set of all prime numbers, $P(X) = \{p \in P \mid T_p(X) \neq 0\}$. All other notions which are used in the paper are standard and can be found, for example, in monographs [4, 5, 9].

Definition 1. A ring R will be called sufficiently π-regular if for each $0 \neq x \in R$ there exist $m \in \mathbb{N}$ and $y \in R$ such that $x^m \neq 0$ and $x^m y x^m = x^m$.

Remark 1. From the definitions of sufficient π-regularity, π-regularity, and regularity of a ring it follows that the class of groups with regular endomorphism ring is contained in the class of groups with sufficiently π-regular endomorphism ring, which in turn is a subclass of the class of groups with π-regular endomorphism ring.

Lemma 1. *For a group G, the following conditions are equivalent*:

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