Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 45, No. 12, pp. 52–64, 2001 *UDC 517.977*

ITERATION PROCEDURES FOR SOLVING PROBLEMS OF OPTIMAL CONTROL ON THE BASIS OF QUASIGRADIENT APPROXIMATIONS

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As before, in the problems of optimal control the gradient methods (procedures of weak variation) keep their actuality and remain to be a reliable mean for a numerical solving (see [1]–[7]). Nevertheless, possibilities for their improvement and modification within the framework of certain classes of problems are yet far from being exhausted.

In this article we suggest a series of modifications of standard gradient procedures (gradient method with constant step, method of gradient's projection) on the basis of more qualitative approximations of the objective functional. Preserving, in the whole, a typical structure of the above methods, new procedures are preferable by virtue of the following characteristics:

- in problems without constraints upon the control, a special function of variation is used (instead of a constant), which leads to well-founded dynamical correction of the antigradient's direction;
- in bilinear and quadratic problems a nonlocal improvement of controls is provided, i. e., on every iteration it is not required to solve the problem of search an appropriate value of the parameter of variation;
- in the nonconvex problems an outstanding possibility for improvement of stationary controls arises due to a discontinuous character of the variation procedure, which brings us new outlooks in search for global solutions;
- in the convex quadratic problem the projection method realizes a minimizing sequence of controls for any value of the parameter of variation.

We substantiate the constructed procedures as concerns the verification of properties of improvability and the proof of convergence for certain classes of problems.

1. Statement of problem. Approximation of functional

Let us define the main problem of optimal control by the relations

$$\text{Problem (P):}\quad \Phi(u) = \varphi(x(t_1)) + \int_T F(x,u,t)dt \to \min, \tag{1}$$

$$\dot{x} = f(x,u,t), \quad x(t_0) = x^0, \tag{2}$$

$$u(t) \in U, \quad t \in T, \tag{3}$$

where $t \in T = [t_0, t_1]$ is an independent variable, $u(t) \in R^r$ a control, $x(t) \in R^n$ a phase state. Suppose that in Problem (P)

Supported by the Russian Foundation for Basic Research (projects nos. 99-01-00400, 00-01-81130) and the Program "UniRos — Fundamental Research" (project no. 990345).