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The Existence of a Linear Horseshoe of Continuous Maps of Dendrites

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Abstract—Assume that a continuous map f defined on a dendrite X has a horseshoe (A, B), where A and B are nonempty disjoint subcontinua in X. In this paper we obtain conditions for the structure of sets A and B under which some iteration of f has a linear horseshoe.

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1. INTRODUCTION

Let X be a continuum (a compact connected metric space) and let $f : X \to X$ be a continuous map. We say that *f has a horseshoe* if there exist disjoint subcontinua $A, B \subset X$ such that

$$f(A) \cap f(B) \supset A \cup B. \tag{1}$$

We denote a horseshoe of a continuous map f by (A, B).

It is well-known that if some iteration of a map f has a horseshoe, then the topological entropy of f is positive (e.g., [1]).

Let X be an one-dimensional continuum. We say that a *map* $f : X \to X$ *has a linear horseshoe* if f has a horseshoe (A, B), where A and B are arcs, i.e., A and B are homeomorphic to a segment of the straight line.

In [2] one demonstrates that for a continuous map f given on a graph (a one-dimensional compact connected polyhedron) the positivity of the topological entropy of f is equivalent to the existence of a linear horseshoe for some iteration of f. In [3, 4] one constructs examples of continuous maps on dendrites with a positive topological entropy that have a horseshoe, but no iteration of the map has a linear horseshoe.

In this paper we study conditions for the structure of a horseshoe (A, B) of a continuous map $f : X \to X$ (here X is a dendrite) which guarantee that some iteration of f has a linear horseshoe. Moreover, we define a class of dendrites that admit the existence of a linear horseshoe.

We say that a *dendrite X admits the existence of a linear horseshoe* if for any continuous map $f : X \to X$ having a horseshoe there exists a positive integer number $n \geq 1$ such that f^n has a linear horseshoe.

The arising interest towards the study of dynamic systems on dendrites is connected, for example, with the fact that dendrites appear as Julia sets in complex dynamic systems (e.g., [5], P. 14). On the other hand, dendrites are examples of Peano continua with complex topology structures (e.g., [6], pp. 165–187).

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