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# STABILITY OF CONTROL WITH RESPECT TO PARAMETER

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Consider the system of differential equations of the form

$$\dot{x} = f(t, x, y, \lambda), \tag{1}$$

in which $x \in E_n$, $u \in E_r$, $\lambda \in E_m$, $u$ is the control, $\lambda$ is the parameter, $t \in [\alpha, \beta]$, $E_s$ is an $s$-dimensional vector space, $\alpha$, $\beta$ ($\alpha < \beta$) are some constant numbers. The set $U$ of admissible controls consists of the measurable, bounded on segment $[\alpha, \beta]$, $r$-dimensional vector functions $u$. For any $u \in U$ and any $\lambda \in E_m$, by a solution of system (1), which is defined on a certain segment, we shall understand a vector function absolutely continuous and almost everywhere on this segment satisfying system (1).

**Definition 1.** Let $x_0$, $x_1$ be points of the space $E_n$. System (1) for $\lambda = \overline{\lambda}$ is called controllable at the points $x_0$, $x_1$ (at the point $x_0$ under the condition $x_0 = x_1$) if a control $\overline{u} \in U$ exists such that system (1) for $u = \overline{u}$, $\lambda = \overline{\lambda}$ possesses a solution $x(t)$, defined on the segment $[\alpha, \beta]$ and satisfying the boundary value conditions

$$x(\alpha) = x_0, \quad x(\beta) = x_1. \tag{2}$$

For the sake of brevity, in what follows system (1) will be called controllable if it satisfies Definition 1.

In this article we pose the following problem: Determine the conditions for preservation (loss) of the controllability of system (1) under change of the parameter in the assumption that system (1) is controllable for a certain value of the parameter. To this end we introduce the notion of stability (instability) of the control with respect to parameter. The parameter $\lambda$ can be treated as the estimate of the influence of the external effects on a controllable system. A similar problem was studied in [1], [2], where one of the requirements was the complete controllability of the system of linear approximation of system (1). Basic results of the article are obtained without assumption of complete controllability of the system of linear approximation. We investigate the cases where system (1) does not possess a linear approximation system at all.

In the general case, not all of coordinates of the vector $\lambda$ can affect the controllable system. Therefore we introduce the functions $\varphi$ and $\psi$ of the natural argument, which determine the numbers of coordinates of the vector $\lambda$, which affect the controllable system, and the numbers of coordinates of the vector $\lambda$, which have no influence on the system.

A vector function definite on the segment $[\alpha, \beta]$ and satisfying with some $u \in U$ and $\lambda \in E_m$ equalities (1), (2) will be called a solution of problem (1), (2). We denote by $x(t, x_0, u, \lambda)$ a solution of system (1), which satisfies the condition $x(\alpha, x_0, u, \lambda) = x_0$, $u \in U$, $\lambda \in E_m$.

Let $x(t, x_0, u_0, \lambda_0)$ be a solution of problem (1), (2). Set

$$z = x - x(t, x_0, u_0, \lambda_0), \quad \vartheta = u - u_0, \quad \mu = \lambda - \lambda_0. \tag{3}$$