ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2010, Vol. 54, No. 2, pp. 33–45. © Allerton Press, Inc., 2010.
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The Riesz Bases Consisting of Eigen and Associated Functions for a Functional Differential Operator with Variable Structure

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Received October 3, 2007

Abstract—We consider a functional differential operator with variable structure with an integral boundary condition. We prove that its eigen and associated functions form a Riesz basis with brackets in the space $L_2^3[0,1]$.

DOI: 10.3103/S1066369X10020052

Key words and phrases: *Riesz basis, resolvent, eigenfunction, boundary condition, regularity.*

Consider a functional differential operator in the form

$$Ly = l[y] = \alpha_j y'(x) + \beta_j y'(\gamma_{j-1} + \gamma_j - x) + p_{j1}(x)y(x) + p_{j2}(x)y(\gamma_{j-1} + \gamma_j - x), \tag{1}$$
$$x \in [\gamma_{j-1}, \gamma_j] \quad (j = 1, 2, 3), \quad 0 = \gamma_0 < \gamma_1 < \gamma_2 < \gamma_3 = 1,$$
$$\int_0^1 y(t)\, d\sigma(t) = 0. \tag{2}$$

Below we impose restrictions on its parameters.

In this paper we continue to study functional differential and integral operators with reflection operators. This area of research has been intensively developed over many years (see [1–7]). We study the Riesz bases with brackets consisting of eigen and associated functions (e. a. f.) of operator (1)–(2). Great achievements in solving this important problem belong to Russian mathematicians ([8–12]). The case of differential and integrodifferential operators with integral boundary conditions is studied in [13, 14].

It is convenient to replace segments $[\gamma_{j-1}, \gamma_j]$ with $[0, 1]$ and to reduce (1)–(2) in the evident way to the following operator in the space of vector functions of dimension 3:

$$Ly = l[y] = \begin{pmatrix} \alpha_1 y_1'(x) + \beta_1 y_1'(1-x) + p_{11}(x)y_1(x) + p_{12}(x)y_1(1-x) \\ \alpha_2 y_2'(x) + \beta_2 y_2'(1-x) + p_{21}(x)y_2(x) + p_{22}(x)y_2(1-x) \\ y_3'(x) + p(x)y_3(x) \end{pmatrix}, \tag{3}$$
$$y_1(0) = y_3(1), \quad y_2(1) = y_3(0), \tag{4}$$
$$\int_0^1 y_1(t)\, d\sigma_1(t) + \int_0^1 y_2(t)\, d\sigma_2(t) + \int_0^1 y_3(t)\, d\sigma_3(t) = 0. \tag{5}$$

Here $y(x) = (y_1(x), y_2(x), y_3(x))^\top$ ($\top$ is the symbol of transposition), α_j, β_j, and $p_{ij}(x)$ have a new sense. On a certain segment $[\gamma_{j-1}, \gamma_j]$ we have a pure differential operator of the first order (for definiteness, we do on the segment $[\gamma_2, \gamma_3]$). Assume that in (3) $\alpha_j^2 \neq \beta_j^2$, $\beta_j \neq 0$, $p(x), p_{ij}(x) \in C^1[0,1]$ $(i, j = 1, 2)$, while $\sigma_j(x)$ $(j = 1, 2, 3)$ are functions of bounded variation with jumps at the points 0 and 1.

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