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THE ADAPTIVE APPROXIMATION OF FUNCTIONS

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A problem of adaptive approximation of functions from L_2 is posed. A method of successive approximations for solving the problem is suggested. Theorems on sufficient conditions for convergence of approximate solution to the desired solution are proved.

1. Statement of problem

Polynomial approximation of functions has as its objective the creation of additional possibilities of their approximate analytical investigation and practical realization. This is attained by application, in the capacity of coordinate functions, of approximations of well-studied expressions and also numerical parameters, i. e., coefficients of the polynomials (see [1], Chap. 3, § 7, p. 42). In this situation it is assumed that the computability of the parameters guarantees their physical realizability.

But the latter fails to take place always. In particular, in a great body of cases, the desired optimal values of approximation parameters practically can be obtained only by successive refinements (corrections) of their initial states with the use of consecutively stored measured information on the approximating functions. Situations of that kind are frequently met and they do not require special investigation. (see [2], p. 135).

In this connection let us consider the problem on optimal approximation of a real function $x^*(t) \in L_2$ on the interval T of values of the real argument t in the class of approximating functions of the form

$$x(t) = U^T u + V^T v + W^T w + \theta^T \vartheta \quad (t \in T). \tag{1.1}$$

Here $U^T = \|U^1(t), \dots, U^n(t)\|$, $V^T = \|V^1(t), \dots, V^m(t)\|$, $W^t = \|W^1(t), \dots, W^r(t)\|$, $\theta^T = \|\theta^1(t), \dots, \theta^q(t)\|$ are line matrices of coordinate functions from L_2; $u = \|u^1, \dots, u^n\|^T$, $v = \|v^1, \dots, v^m\|^T$, $w = \|w^1, \dots, w^r\|^T$, $\vartheta = \|\vartheta^1, \dots, \vartheta^q\|^T$ are column matrices of approximation parameters; T is the matrix transposition symbol.

By optimal one we shall call an approximation

$$x_f(t) = U^T u_f + V^T v_f + W^T w_f + \theta^T \vartheta, \tag{1.2}$$

whose parameters $u_f = \|u_f^1, \dots, u_f^n\|^T$, $v_f = \|v_f^1, \dots, v_f^m\|^T$ supply the minimum to the function

$$Y = \int_T [x(t) - x^*(t)]^2 dt \tag{1.3}$$

by variables u, v under condition $w = w_f = \|w_f^1, \dots, w_f^r\|^T$, where w_f is a given matrix.

Obviously, u_f and v_f must be defined via the equations (see [1], Chap. 3, § 7, p. 43)

$$\begin{aligned} (U, U^T)u_f + (U, V^T)v_f &= (x^*, U) - (U, W^T)w_f - (U, \theta^T)\vartheta, \\ (V, U^T)u_f + (V, V^T)v_f &= (x^*, V) - (V, W^T)w_f - (V, \theta^T)\vartheta. \end{aligned} \tag{1.4}$$