ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2015, Vol. 59, No. 4, pp. 63–70. © Allerton Press, Inc., 2015.
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Sequences of Non-Uniqueness for Weight Spaces of Holomorphic Functions

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Received October 6, 2013

Abstract—Problems of description of zero subsequences (non-uniqueness sequences) for weight spaces of holomorphic functions are reduced, according to a general scheme, to solving certain problems in weight classes of subharmonic functions. We also mention geometrical aspects of the topic and completeness of exponential systems.

DOI: 10.3103/S1066369X1504009X

Keywords: *holomorphy, zero subsequence, non-uniqueness sequence, subharmonic function, exponential system, ρ-convex complementability, geometric difference, full sweeping-out.*

1. INTRODUCTION

1.1. Basic designations, definitions, and agreements. As usual, we mean by $\mathbb{R}$ and $\mathbb{C}$ the sets of all *real* and *complex numbers* as long as their geometric interpretations. We denote by $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$ the *unit disk* in the complex plane $\mathbb{C}$. Further we will use definitions and notation from [1]. Let D be a domain in $\mathbb{C}$. We associate with every at most countable sequence $\Lambda = \{\lambda_k\}_{k\geq 1} \subset D$, without limiting points in D, the *counting measure* $n_\Lambda(S) := \sum\limits_{\lambda_k \in S} 1$, i.e., the number of points from Λ lying in $S \subset D$. We do not exclude that among the points λ_k can be repeated ones. The union $\Lambda \cup \Lambda_0$ of two such sequences $\Lambda, \Lambda_0 \subset D$ is completely determined by the counting measure $n_{\Lambda\cup\Lambda_0} := n_\Lambda + n_{\Lambda_0}$. By definition, the function $n_\Lambda(\lambda) := n_\Lambda\bigl(\{\lambda\}\bigr)$ is the divisor of the sequence Λ, i.e., the number of attendance of $\lambda \in \mathbb{C}$ in Λ. Thus, $\lambda \in \Lambda$ if $n_\Lambda(\lambda) > 0$.

We denote by $\mathrm{Hol}(D)$ the vector space of all holomorphic in D functions. Unless otherwise stated, we endow the space by the topology of locally uniform convergence. We juxtapose to a nonzero $f \in \mathrm{Hol}(D)$ its *zero sequence* Zero_f renumbered with account taken of multiplicities of its zeros.

For a compact $C \subset \mathbb{C}$ we denote by $\mathrm{CHol}[C]$ the vector space (over $\mathbb{C}$) of all continuous on C complex-valued functions which are holomorphic in the *interior* $\mathrm{int}\, C$ of C, if it is non-empty, with the natural sup-norm.

A sequence $\Lambda \subset D$ is called a *zero subsequence* for a subset $H \subset \mathrm{Hol}(D)$, if there exists a nonzero function $f \in H$ such that $\Lambda \subset \mathrm{Zero}_f$ in the sense that $n_\Lambda(\lambda) \leq n_{\mathrm{Zero}_f}(\lambda)$ for all $\lambda \in D$. If H is closed with respect to the difference operator, for example, if it is a vector space over $\mathbb{R}$, then a zero subsequence for H is called a *non-uniqueness sequence* (or a *non-uniqueness set*) for H.

We denote by $\mathrm{sbh}(D)$ the convex cone of all subharmonic functions in a domain $D \subset \mathbb{C}$ and by $-\boldsymbol{\infty}$ the subharmonic function identically equal $-\infty$ on D. For $s \in \mathrm{sbh}(D)$ we will mostly denote the Riesz measure of s by ν_s, and, vice versa, we will mostly denote the subharmonic function s in D with the Riesz measure ν as $s := s_\nu$. A Borel positive measure (finite on compacts in D) or a Radon measure ν ([2], Appendix A) is called the *submeasure for a subset* $S \subset \mathrm{sbh}(D)$, if there exists a function $s \in S$, distinct from $-\boldsymbol{\infty}$, with the Riesz measure $\nu_s \geq \nu$ on D. In other words, ν is the submeasure for S, if for some (every) subharmonic function s_ν with the Riesz measure ν there exists a function $v \in \mathrm{sbh}(D)$, distinct from $-\boldsymbol{\infty}$, such that $s := s_\nu + v \in S$. Possibility of interchange of the words “some” and “every” in the

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