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# Upper Bound of Unreliability of Circuits in a Basis Consisting of Webb Functions

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**Abstract**—We consider a realization of ternary logics functions by the circuits with unreliable functional gates in the basis which consists of the Webb function. It is assumed that all gates are supposed to have faults on the outputs and they go to the faulty states independently. We present a method for the synthesis of reliable circuits and an upper bound for the unreliability of these circuits.

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In modern Mathematics and Technics the theory of synthesis of circuits (see the definition of scheme, for example, in [1], P. 10) composed of unreliable functional gates is important. It is worthy to note that circuits dominate that are functioning on the base of two-valued logic. However, the complexity of solved problem and therefore, the complexity of technical systems, permanently increase. Many technologic possibilities draw to a close, such as the increase of elements density on circuits and the augmentation of work frequency. The use of multivalued logic is one of ways for the solution of formulated problems.

The multivalued logic provides wider possibilities for elaboration of different algorithms in many areas. It allows to decrease the computational complexity and dimensions, the number of connections in different arithmetic-logic systems, increase the density of elements on circuits, find alternative methods of solving the problems.

One repeatedly appeals to multivalued logic, to their mathematical apparatus as a source of mathematical models, which have great potential possibilities [2, 3]. In [4] one has constructed a functionally complete in $P_3$ basis, in which on compromise base one has coordinated mathematical and technical (MDS-techniques originates from the phrase "metal, dielectric, and semiconductor") requirements and interests, and consider some aspects of synthesis of electronic circuits in this basis.

The problem of reliability investigation of circuits functioning in a complete basis of three-valued functions has a certain interest. We note that the multivalued synthesis has a series of singularities, and the question about the reliability of circuits, which realize the functions of three-valued logic, is poorly understood to the present moment. To our knowledge, the upper estimate for the reliability of schemes in the Rosser–Turkett basis has been received in [5]. In this paper we propose the method of synthesis of reliable circuits in a complete basis composed of the Webb function $V_3(x_1, x_2) = \max(x_1, x_2) + 1 (\mathrm{mod}\ 3)$.

Let $n \in \mathbb{N}$, and $P_3$ be a set of all functions of three-valued logic, i.e., functions $f(x_1, \ldots, x_n) : \{0, 1, 2\}^n \to \{0, 1, 2\}$. Denote by $\widetilde{x}$ the tuple $(x_1, \ldots, x_n)$.

Let us consider the realization of functions from the set $P_3$ by circuits composed of unreliable functional elements in the base composed of the Webb function $V_3(x_1, x_2) = \max(x_1, x_2) + 1 (\mathrm{mod}\ 3)$ (further we assume that all operations are taken modulo 3). The completeness of this set in $P_3$ has been proved, for example, in [6] (P. 50).

We assume that the circuit composed of unreliable elements realizes the system of functions $f_1(\widetilde{x}), \ldots, f_l(\widetilde{x})$, $l \in \mathbb{N}$, if with entering of the tuple $\widetilde{a}$ on the inputs of circuit without faults in circuit

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