Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 42, No. 5, pp. 66–70, 1998 *UDC 517.537*

ABOUT THE MAXIMAL TERM OF THE DERIVATIVE OF THE DIRICHLET SERIES

M.N. Sheremeta

1. Let $0=\lambda_0<\lambda_n\uparrow+\infty$, $n\to\infty$, the Dirichlet series

$$F(s)=\sum_{n=0}^{\infty}a_n\exp(s\lambda_n),\quad s=\sigma+it, \tag{1}$$

have an abscissa of absolute convergence $A\in(-\infty,+\infty]$, and $\mu(\sigma)=\mu(\sigma,F)=\max\{|a_n|\exp(\sigma\lambda_n), n\ge 0\}$ be the maximal term of series (1). We denote by $\Omega(A)$ a class of positive unbounded on $(-\infty,A)$ functions Φ such that the derivative Φ' is positive, continuous, and grows to $+\infty$ on $(-\infty,A)$. Let φ be a function inverse to Φ', and $\Psi(\sigma)=\sigma-\Phi(\sigma)/\Phi'(\sigma)$ be a function associated with Φ by Newton. Then the function φ is continuous on $(0,+\infty)$ and increases to A while the function Ψ is continuous on $(-\infty,A)$ and also increases to A. For the case $A=+\infty$, the latter property was proved in [1], one can analogously prove it for the case where $A\in(-\infty,+\infty)$.

The main result of the present article is the following

Theorem 1. *Let $A\in(-\infty,+\infty]$, $\Phi\in\Omega(A)$, the Dirichlet series* (1) *have the abscissa of absolute convergence A, and*

$$\varlimsup_{\sigma\to A}\frac{\ln\mu(\sigma,F)}{\Phi(\sigma)}=1. \tag{2}$$

Then

$$\varlimsup_{\sigma\to A}\frac{\mu(\sigma,F')}{\mu(\sigma,F)\Phi'(\sigma)}\ge 1 \tag{3}$$

and if we have, in addition,

$$\ln\Phi'(\sigma)=o(\Phi(\sigma)),\quad \sigma\to A, \tag{4}$$

then

$$\varlimsup_{\sigma\to A}\frac{\mu(\sigma,F')}{\mu(\sigma,F)\Phi'(\Psi^{-1}(\sigma))}\le 1. \tag{5}$$

The exactness of estimate (3) is undoubtful. The following theorem shows the unimprovability of estimate (5).

Theorem 2. *Let $A\in(-\infty,+\infty]$, $\Phi\in\Omega(A)$, and*

$$F_0(s)=\sum_{n=0}^{\infty}\exp\{-\lambda_n\Psi(\varphi(\lambda_n))+s\lambda_n\}.$$