

## ABOUT THE MAXIMAL TERM OF THE DERIVATIVE OF THE DIRICHLET SERIES

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1. Let  $0 = \lambda_0 < \lambda_n \uparrow +\infty$ ,  $n \rightarrow \infty$ , the Dirichlet series

$$F(s) = \sum_{n=0}^{\infty} a_n \exp(s\lambda_n), \quad s = \sigma + it, \quad (1)$$

have an abscissa of absolute convergence  $A \in (-\infty, +\infty]$ , and  $\mu(\sigma) = \mu(\sigma, F) = \max\{|a_n| \exp(\sigma\lambda_n), n \geq 0\}$  be the maximal term of series (1). We denote by  $\Omega(A)$  a class of positive unbounded on  $(-\infty, A)$  functions  $\Phi$  such that the derivative  $\Phi'$  is positive, continuous, and grows to  $+\infty$  on  $(-\infty, A)$ . Let  $\varphi$  be a function inverse to  $\Phi'$ , and  $\Psi(\sigma) = \sigma - \Phi(\sigma)/\Phi'(\sigma)$  be a function associated with  $\Phi$  by Newton. Then the function  $\varphi$  is continuous on  $(0, +\infty)$  and increases to  $A$  while the function  $\Psi$  is continuous on  $(-\infty, A)$  and also increases to  $A$ . For the case  $A = +\infty$ , the latter property was proved in [1], one can analogously prove it for the case where  $A \in (-\infty, +\infty)$ .

The main result of the present article is the following

**Theorem 1.** *Let  $A \in (-\infty, +\infty]$ ,  $\Phi \in \Omega(A)$ , the Dirichlet series (1) have the abscissa of absolute convergence  $A$ , and*

$$\overline{\lim}_{\sigma \rightarrow A} \frac{\ln \mu(\sigma, F)}{\Phi(\sigma)} = 1. \quad (2)$$

*Then*

$$\overline{\lim}_{\sigma \rightarrow A} \frac{\mu(\sigma, F')}{\mu(\sigma, F)\Phi'(\sigma)} \geq 1 \quad (3)$$

*and if we have, in addition,*

$$\ln \Phi'(\sigma) = o(\Phi(\sigma)), \quad \sigma \rightarrow A, \quad (4)$$

*then*

$$\overline{\lim}_{\sigma \rightarrow A} \frac{\mu(\sigma, F')}{\mu(\sigma, F)\Phi'(\Psi^{-1}(\sigma))} \leq 1. \quad (5)$$

The exactness of estimate (3) is undoubtful. The following theorem shows the unimprovability of estimate (5).

**Theorem 2.** *Let  $A \in (-\infty, +\infty]$ ,  $\Phi \in \Omega(A)$ , and*

$$F_0(s) = \sum_{n=0}^{\infty} \exp\{-\lambda_n \Psi(\varphi(\lambda_n)) + s\lambda_n\}.$$

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