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STATEMENT AND INVESTIGATION OF STATIONARY PROBLEM ON CONTACT OF A SOFT SHELL AND A BARRIER

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Introduction

We consider the problem on determination of the equilibrium state of a soft shell fixed on edges, being under action of mass and surface loads, and restricted in its motion by a barrier. Finite deformations and increments of the shell are admitted.

A plane version of the problem on equilibrium state of a thread under the action of load restricted in its position by a halfplane was posed in the form of a variational inequality in [1]. In the same paper the existence of the solution of this problem was established. The case where the boundary of the barrier is a concave function was considered for the plane problem in [2]. In [3] for the same problem the case was considered where the set of admissible configurations is arbitrary.

In this article we consider the spatial problem on equilibrium state of a soft shell under the condition that the surface of the barrier is described by a sufficiently smooth (not necessarily convex) function. First, starting from the equilibrium equations written in the Cartesian coordinate system, we formulate a pointwise problem. Afterwards, on the basis of the principle of virtual displacements, we obtain its variational formulation. We establish the equivalence of the mentioned problems.

In the second part of the article we consider the case of a soft netlike shell formed by two families of threads. Under certain conditions upon the functions describing physical relations in the threads, we state a generalized problem in the form of a quasivariational inequality in a Banach space and establish its solvability. Note that the corresponding problem under absence of a barrier was studied in [4].

1. Pointwise and variational formulations of the problem

We introduce the Cartesian coordinate system (x_1, x_2, x_3). Assume that in a non-deformed state the shell can be described by the surface $\xi(\alpha) = (\xi_1(\alpha), \xi_2(\alpha), \xi_3(\alpha))$, where $\alpha = (\alpha_1, \alpha_2) \in \Omega$ are the Lagrange coordinates, Ω is a bounded domain from R^2 with a Lipschitz-continuous boundary Γ; we assume that the function ξ satisfies the conditions

$$\xi \in [C_1(\overline{\Omega})]^3, \quad |[\partial_1\xi(\alpha), \partial_2\xi(\alpha)]| \geq c > 0 \quad \forall \alpha \in \overline{\Omega}. \tag{1}$$

We denote by $w(\alpha) = (w_1(\alpha), w_2(\alpha), w_3(\alpha))$ a function describing the surface of the shell in its deformed state; $G(\alpha) = |[\partial_1 w(\alpha), \partial_2 w(\alpha)]|^2$ is the discriminant of the metric tensor of the surface of the deformed shell.

Here we have used the notation $\partial_k = \partial/\partial\alpha_k$, $k = 1, 2$; $[\cdot,\cdot]$, $(\cdot,\cdot)$, and $|\cdot|$ are the vector, scalar products, and the norm in R^3, respectively.

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